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2020 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2020, Virtual, November 16-19, 2020

Parameter Estimation of Nonlinear Circuits using Haar Wavelet Transform

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Abstract– There have been proposed several methods to analyze waveforms in transient state and waveforms in steady state of nonlinear circuits using Haar wavelet transforms [1], [2]. Using the Haar wavelet, derivative and integration operation matrices were easily obtained and they can be applied to solve ordinary differential equations [3], [4]. On the other hand, estimating unknown parameters of nonlinear circuits from observed waveforms is important in system identification problems. Therefore, the purpose of this study is to estimate the parameters of the unknown nonlinear circuit from the steady state waveform and the transient state waveform by using the Haar wavelet as inverse problem of analyses of the waveforms nonlinear ODEs.

1. Introduction

In recent years, attention has been paid to the solving methods of differential equations using wavelet transform. We have proposed the methods for analyzing a transient state waveform and a steady state waveform of a nonlinear circuit by using Haar wavelet transforms [1], [2]. In addition, it is considered that more accurate parameter estimation is possible from the orthogonality/locality of wavelets. Among the wavelet transforms, the Haar wavelet transform has attracted particular attention, because derivative and integral operation matrices by using block pulse functions can be easily obtained [3], [4] and they can be applied to the differential equations.

On the other hand, estimating the unknown parameters of the nonlinear circuit from the observed waveforms is important in the system identification problem. For such problems, we consider we can apply the Haar wavelet transforms to such problems as inverse problems of waveform analyses described above.

Therefore, the purpose of this research is to estimate the parameters of an unknown nonlinear circuit from the waveform of the steady state and the waveform of the transient state by using the Haar wavelet as we consider the inverse problem of solving ODEs. By using some examples, we can obtain the accurate parameter estimation from the proposed method.

2. Haar Wavelet Matrix

The Haar function h_i is defined in the interval [0,1) as follows.

$$h_0 = \frac{1}{\sqrt{m}} \tag{1}$$

$$h_i = \frac{1}{\sqrt{m}} \times \begin{cases} 2^{\frac{j}{2}}, & \frac{k-1}{2^j} \le t < \frac{k-\frac{1}{2}}{2^j}, \\ -2^{\frac{j}{2}} & \frac{k-\frac{1}{2}}{2^j} \le t < \frac{k}{2^j}, \\ 0 & \text{otherwise in } [0,1) \end{cases} \tag{2}$$

Then $i = 0, 1, \cdots, m-1, m = 2^{\alpha}$, $i = 2^j + k$,i.e., $k = 0, 1, \cdots, 2^j - 1$ $(j = 0, 1, 2, \cdots)$.

The Haar wavelet matrix H is an $m \times m$ order matrix defined by the Haar function h_i.

$$H = \left[\overrightarrow{h_0^T} \ \ \overrightarrow{h_1^T} \cdots \overrightarrow{h_{m-1}^T}\right] \tag{3}$$

By using this matrix, the Haar wavelet transform and the inverse Haar wavelet transform are respectively expressed as follows.

$$\vec{c} = H\vec{y} \tag{4}$$

$$\vec{y} = H^T\vec{c} (= H^{-1}\vec{c}) \tag{5}$$

The integral operational matrix of the block pulse function matrix B is defined by the following equation. In this case, it is assumed that $B(t)$ is displayed in a $(m \times m)$-order matrix by discrete time expression.

$$\int_0^i B(\tau)d\tau \equiv Q_B \cdot B(t) \tag{6}$$

$$Q_{B(m\times m)} = \frac{1}{m}\left[\frac{1}{2}I_{(m\times m)} + \sum_{i=1}^{\infty} P^i_{(m\times m)}\right] \tag{7}$$

$$P^i_{(m\times m)} = \begin{bmatrix} 0 & I_{(m-i)\times(m-i)} \\ 0_{(i\times i)} & 0 \end{bmatrix} \text{ for } i < m \tag{8}$$

$$P^i_{(m\times m)} = 0_{(m\times m)} \ \text{ for } \ i \ge m \tag{9}$$

The integral and differential matrix using the Haar wavelet matrix H are given as follows, respectively,

$$Q_H = HQ_B^T H^{-1} = HQ_B^T H^T \tag{10}$$

$$Q_H^{-1} = H(Q_B^T)^{-1}H^{-1} = H(Q_B^T)^{-1}H^T \tag{11}$$

3. Haar Wavelet Expression of Branch Characteristics

The Haar wavelet representation of nonlinear time varying circuit elements such as capacitors, inductors and resistors can be expressed as follows [1], [2].

Capacitor:

$v(t) = v(0_-) + \frac{1}{C}\int_0^t i(\tau)d\tau$

$V = V_0 + C_w^{-1}Q_H I$ or $I = C_w Q_H^{-1}[V - V_0]$

$C_w = H\mathrm{diag}[C(i_0,t_0), C(i_1,t_1), \cdots, C(i_{m-1},t_{m-1})]H^T$ (12)

Inductor:

$i(t) = i(0_-) + \frac{1}{L}\int_0^t v(\tau)d\tau$

$i(t) = i(0_-) + \frac{1}{L}\int_0^t v(\tau)d\tau$ or $V = Q_H^{-1}L_w[I - I_0]$

$L_w = H\mathrm{diag}[L(i_0,t_0), L(i_1,t_1), \cdots, L(i_{m-1},t_{m-1})]H^T$ (13)

Resistance:

$v(t) = Ri(t)$

$V = R_w I,\ R_w = \mathrm{diag}[R]$

$R_w = H\mathrm{diag}[R(i_0,t_0), R(i_1,t_1), \cdots, R(i_{m-1},t_{m-1})]H^T$ (14)

4. Parameter Estimation Method for Nonlinear Circuits

4.1. How to Find the Parameter Matrix

Consider the following differential equation.

$$\dot{x} = f(x,t) \triangleq A(x,t)x + u(t) \tag{15}$$

$x(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T \in R^{n\times 1}$ is a known state variable vector, $A(x,t) = \mathrm{diag}(a_1(x,t)\ a_2(x,t)\ \cdots\ a_n(x,t)) \in R^{n\times n}$ is an unknown nonlinear time-varying parameter matrix, and $u(t) = [u_1(t)\ u_2(t)\ \cdots\ u_n(t)]^T \in R^{n\times 1}$ is an external force vector. Also, $A(x,t)$ is a diagonal matrix.

At this time, discrete expressions of $x(t)$, $u(t)$, and $A(x,t)$ are defined as follows.

$$\overrightarrow{x_i} = [x_i(t_1)\ x_i(t_2)\ \cdots\ x_i(t_m)]^T \in R^{m\times 1} \quad \text{for } i = 1,2,\cdots,m \tag{16}$$

$$\overrightarrow{u_i} = [u_i(t_1)\ u_i(t_2)\ \cdots\ u_i(t_m)]^T \in R^{m\times 1} \quad \text{for } i = 1,2,\cdots,m \tag{17}$$

$$\overrightarrow{A_i} = \mathrm{diag}\big(a_i(t_1)\ a_i(t_2)\ \cdots\ a_i(t_m)\big) \in R^{m\times m} \quad \text{for } i = 1,2,\cdots,m \tag{18}$$

The initial value vector is defined as follows.

$$\overrightarrow{x_{i0}} = [x_i(0)\ x_i(0)\ \cdots\ x_i(0)]^T \in R^{m\times 1} \tag{19}$$

$$\overrightarrow{x_0} = [x_1(0)\ x_2(0)\ \cdots\ x_n(0)]^T \ \text{for } i = 1,2,\cdots,m \tag{20}$$

The wavelet expression in Eq. (15) is derived as in Eq. (21).

$$Q_m^{-1}[X - X_0] = A_H X + U$$
$$A_H X = Q_m^{-1}[X - X_0] - U \tag{21}$$

Then,

$$X = [(H\overrightarrow{x_1})^T\ (H\overrightarrow{x_2})^T\ \cdots\ (H\overrightarrow{x_n})^T]^T \triangleq [X_1^T\ X_2^T\ \cdots\ X_n^T]^T \in R^{mn\times 1} \tag{22}$$

$$X_0 = [(H\overrightarrow{x_{10}})^T\ (H\overrightarrow{x_{20}})^T\ \cdots\ (H\overrightarrow{x_{n0}})^T]^T \triangleq [X_{10}^T\ X_{20}^T\ \cdots\ X_{n0}^T]^T \in R^{mn\times 1} \tag{23}$$

$$U = [(H\overrightarrow{u_1})^T\ (H\overrightarrow{u_2})^T\ \cdots\ (H\overrightarrow{u_n})^T]^T \in R^{mn\times 1} \tag{24}$$

$$A_H = \mathrm{diag}\Big((H\overrightarrow{A_1})(H\overrightarrow{A_2})\cdots(H\overrightarrow{A_n})\Big) \triangleq \mathrm{diag}\big(A_{H_1}\ A_{H_2}\ \cdots\ A_{H_n}\big) \in R^{mn\times mn} \tag{25}$$

4.2. Solving Method using Diagonal Matrix

Consider the solution of Eq. (21) when $A(x,t)$ is defined as a diagonal matrix. The left side of Eq. (21) when $i = 1$ can be transformed into the following form.

$$A_H X = \begin{pmatrix} h_{11}x_1(t_1) & \cdots & h_{1m}x_1(t_m) \\ \vdots & \ddots & \vdots \\ h_{m1}x_1(t_1) & \cdots & h_{mm}x_1(t_m) \end{pmatrix}\begin{pmatrix} a_1 \\ \vdots \\ a_1 \end{pmatrix} \tag{26}$$

Then,

$$\begin{pmatrix} h_{11}x_i(t_1) & \cdots & h_{1m}x_i(t_m) \\ \vdots & \ddots & \vdots \\ h_{m1}x_i(t_1) & \cdots & h_{mm}x_i(t_m) \end{pmatrix} = \overrightarrow{hx_i} \tag{27}$$

If we set it to Eq. (27), the left-hand side of any given i is as follows in Eq. (21).

$$A_H X = \begin{pmatrix} \overrightarrow{hx_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \overrightarrow{hx_n} \end{pmatrix}\begin{pmatrix} a_1 \\ \vdots \\ a_1 \\ a_2 \\ \vdots \\ a_2 \\ \vdots \\ a_n \\ \vdots \\ a_n \end{pmatrix} \tag{28}$$

Therefore, Eq. (21) can be finally transformed as follows.

$$\begin{pmatrix} a_1 \\ \vdots \\ a_1 \\ a_2 \\ \vdots \\ a_2 \\ \vdots \\ a_n \\ \vdots \\ a_n \end{pmatrix} = \left\{\begin{pmatrix} \overrightarrow{hx_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \overrightarrow{hx_n} \end{pmatrix}\right\}^{-1} (Q_m^{-1}[X - X_0] - U) \tag{29}$$

By solving Eq. (29), the parameters of the diagonal components of the unknown parameter matrix A can be obtained.

5. Calculation Example

5.1. Circuit Used for Experiment

In this section, we take the van der Pol oscillator of Fig. 1 as an example to demonstrate the effectiveness of the proposed method. Also, it gives a nonlinear characteristic as shown in Eq. (30).

$$F(V_{OUT}) = (g_1 - g_3 V_{OUT}^2) V_{OUT} \quad (30)$$

From Eq. (30) and Kirchhoff's current law, the circuit equation such as Eq. (31) can be obtained.

$$(g_1 - g_3 V_{OUT}^2) V_{OUT} = i_c(t) + i_L(t) \quad (31)$$

The initial values at this case are as follows.

$$v_c(0) = 0[V],\ \ i_L(0) = 1[A] \quad (32)$$

5.2. How to Find Circuit Parameters

Wavelet transform of Eq. (31) gives Eq. (33).

$$G_w V = C_W Q_H^{-1}[V - V_0] + I_{L0} + L_W^{-1} Q_H V \quad (33)$$

Then,

$$G_w = H \mathrm{diag}[g_1 - g_3 v_0^2, \dots, g_1 - g_3 v_{m-1}^2] H^T \quad (34)$$

The left side of Eq. (33) is as follows.

$$G_w V = \begin{pmatrix} h_{11} v_1 & \cdots & h_{1m} v_m \\ \vdots & \ddots & \vdots \\ h_{m1} v_1 & \cdots & h_{mm} v_m \end{pmatrix} \begin{pmatrix} g_1 - g_3 v_0^2 \\ \vdots \\ g_1 - g_3 v_{m-1}^2 \end{pmatrix} \quad (35)$$

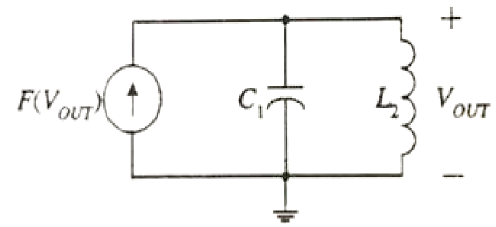

Figure 1 van der Pol oscillator

Therefore, Eq. (33) can be finally transformed as follows.

$$\begin{pmatrix} g_1 - g_3 v_0^2 \\ \vdots \\ g_1 - g_3 v_{m-1}^2 \end{pmatrix} = \left\{ \begin{pmatrix} h_{11} v_1 & \cdots & h_{1m} v_m \\ \vdots & \ddots & \vdots \\ h_{m1} v_1 & \cdots & h_{mm} v_m \end{pmatrix} \right\}^{-1} (C_W Q_H^{-1}[V - V_0] + I_{L0} + L_W^{-1} Q_H V) \quad (36)$$

The unknown parameters g_1 and g_3 can be derived by inputting the transient state v and the steady state v into Eq. (36).

6. Result

6.1. Parameter Estimation under Transient Conditions

In this section, we input waveform of voltage v in the transient state. Input the voltage waveform in the transient state as shown in Fig. 2. In Figure 2, C=0.2, L=0.1, $g_1 = 2.0, g_3 = 1.0$.

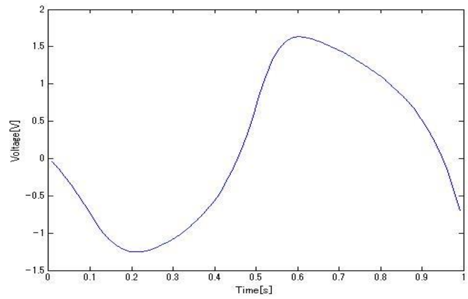

Figure 2 Voltage waveform in transient state when C=0.2, L=0.1, , $g_1 = 2.0, g_3 = 1.0$

Table 1 shows the results when the waveform of voltage v is input when the parameters are $g_1 = 2.0$ and $g_3 = 1.0$.

Table 1 Results when $g_1 = 2.0$ and $g_3 = 1.0$.

C	L	g1	g3	Error of g 1	Error of g3
0.2	0.1	1.992705	0.995097	0.003648	0.004903
0.1	0.2	2.042128	1.022077	0.021064	0.022077
0.2	0.2	1.929054	0.962965	0.035473	0.037035
0.1	0.1	2.204564	1.106231	0.102282	0.106231
0.3	0.2	1.943700	0.961473	0.028150	0.038527
0.2	0.3	2.010552	1.006125	0.005276	0.006125
0.1	0.3	2.039466	1.019802	0.019733	0.019802

Table 2 shows the results when the waveform of voltage v is input when the parameters are $g_1 = 5.0$ and $g_3 = 5/3$.

Table 2 Results when $g_1 = 5.0$ and $g_3 = 5/3$.

C	L	g1	g3	Error of g 1	Error of g3
1	1	4.910031	1.632571	0.017994	0.020457
0.2	0.3	4.722131	1.552014	0.055574	0.068792
0.2	0.2	4.854434	1.610010	0.029113	0.033994
0.1	0.1	4.331756	1.424102	0.133649	0.145539
0.5	0.01	5.021109	1.669028	0.004222	0.001417
0.5	0.05	4.925387	1.636558	0.014923	0.018065
0.6	0.01	5.221641	1.777662	0.044328	0.066597

From Tables 1 and 2, the parameters can be estimated in the transient state accurately.

6.2. Parameter Estimation in Steady State

In this section, we input the waveform of voltage v in steady state. Input a steady-state voltage waveform as shown in Fig. 3. In Figure 3, C=0.2, L=0.1, $g_1 = 2.0, g_3 = 1.0$.

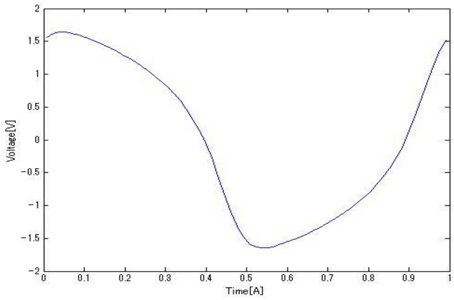

Figure 3 Voltage waveform in transient state when C=0.2, L=0.1, , $g_1 = 2.0, g_3 = 1.0$

Table 3 shows the results when the waveform of voltage v is input when the parameters are $g_1 = 2.0$ and $g_3 = 1.0$.

Table 3 Results when $g_1 = 2.0$ and $g_3 = 1.0$.

C	L	g1	g3	Error of g 1	Error of g3
0.1	0.2	1.785147	0.897957	0.107427	0.102043
0.2	0.1	2.100353	1.050133	0.050177	0.050133
0.2	0.2	2.000052	1.000117	0.000026	0.000117
0.1	0.1	1.956990	0.978975	0.021505	0.021025
0.3	0.2	1.982986	0.991620	0.008507	0.008380
0.2	0.3	2.075559	1.037699	0.037780	0.037699
0.1	0.3	2.026018	1.012670	0.013009	0.012670

Table 4 shows the results when the waveform of voltage v is input when the parameters are $g_1 = 5.0$ and $g_3 = 5/3$.

Table 4 Results when $g_1 = 5.0$ and $g_3 = 5/3$.

C	L	g1	g3	Error of g 1	Error of g3
1	1	4.995796	1.661994	0.000841	0.002804
0.2	0.3	5.039082	1.678339	0.007816	0.007003
0.2	0.2	4.999288	1.666089	0.000142	0.000347
0.1	0.1	4.541309	1.524805	0.091738	0.085117
0.5	0.01	5.255986	1.749351	0.051197	0.049611
0.5	0.05	5.043442	1.681837	0.008688	0.009102
0.6	0.01	5.145399	1.715063	0.029080	0.029038

It is clear from Tables 3 and 4 that the parameters can be estimated in the steady state.

7. Conclusions

In this paper, we proposed a method for estimating unknown nonlinear circuit parameters from transient and steady-state waveforms by considering the inverse problems of waveform analyses using the Haar wavelet transform, and confirmed its operation.

In this paper, we used the van der Pol oscillator, which is one of the nonlinear circuits, as an example, and applied the proposed method. This method uses the fact that when unknown parameters of a non-linear circuit are discretized, the matrix always becomes a diagonal matrix. Therefore, the parameter matrix is a diagonal matrix for the determinant when the circuit equation is wavelet transformed. Unknown parameters can be estimated by performing transformations that can only be used in some cases.

The results show that the unknown parameters of the van der Pol oscillator can be estimated from the transient and steady-state waveforms, so this method is effective as a method to estimate the unknown parameters of the nonlinear circuit from the observed waveforms.

However, depending on the values of C and L, a large error may occur in both the transient state and the steady state. If the period of the waveform is longer than 1 second, a large error will occur. However, the waveform treated in this paper has a large error even though it uses a waveform whose period is smaller than 1 second. Therefore, it is an issue to reduce this error in the future. At the same time, it is a future task to apply this method not only to van der Pol oscillators but also to other nonlinear circuits such as chaotic circuits.

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