A method to find periodic solutions in nonautonomous nonlinear circuits using Haar wavelet transform

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A method to find periodic solutions in nonautonomous nonlinear circuits using Haar wavelet transform

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Abstract: Recently, much attention have been paid to the methods for circuit analysis using wavelet transform. In particular, we have proposed the method which can choose the resolution of the wavelet adaptively. This method can fully bring out the orthogonal and the multiresolution properties of the wavelet, and the efficiency of the calculation can be improved. In this paper, we propose the method to analyze the steady-state periodic solutions of the nonlinear circuits driven by the periodic external input using Haar wavelet transform by applying the appropriate boundary conditions, and prove the effectiveness of the proposed method.

Key Words: Haar wavelet transform, differential equations, nonautonomous systems, numerical calculations, periodic solutions

1. Introduction

The wavelet transform has been often used in signal processing because of its orthogonality and multiresolution property [1, 2]. Recently, much attention has been paid to the method for circuit analysis using wavelet transform [3–9]. The wavelets have the merit to be able to analyze the trajectory near the singular points where the trajectory moves rapidly with high resolution because of the orthogonality and localization property of the wavelet functions. As circuit analysis methods using this merit, some methods were proposed to pick out the ranges automatically where the trajectory moves rapidly near singular points. Thus we have proposed the method for transient circuit analysis using wavelet transform with adaptive resolutions [6]. In this method, the result of the multiresolution analysis is used to choose the range to be analyzed more precisely. It makes the adaptive choice of the wavelet resolution possible, and as a result, the efficient calculation can be achieved.

Moreover, Barmada et al. have proposed the Fourier-like approach for the circuit analysis using the wavelet transform [7]. In this method, the integral and differential operator matrices are introduced to the analysis, and the differential and integral equations are transformed into the algebraic equations like as using Fourier or Laplace transforms. Moreover, the method can treat time varying and nonlinear circuits. Therefore, this method is useful for various circuit analyses. However, in that method, the use of Daubechies wavelet makes the handling of the operator matrices complicated, especially,

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in the edges of the interval. Thus, we have proposed the circuit analysis method using Haar wavelet transform [8]. The Haar wavelet is easy to handle itself, and the operator matrices using the Haar wavelets are easily derived by introducing the block pulse functions [10, 11]. Moreover, the proposed method can treat the nonlinear time varying circuits.

On the other hand, the steady state analysis of the nonlinear circuits is quite important problem in circuit analysis. If we calculate such steady-state waveforms using time-marching methods as in the conventional way, the calculation cost is wasted due to the calculation of the long-term transient response with sufficiently small step size to approximate the discontinuous dynamics typically seen in power electronic circuits. To overcome such disadvantage of the time-marching method, the wavelet method to analyze the steady-state waveforms for power electronics circuits have proposed by Tam et. al [9]. In [9], the Chebyshev polynomials are used as the basis functions for wavelet approach, and the periodic solutions of periodically driven power electronics circuits have been calculated. However it is considered that the calculation should be complicated and the Gibbs-phenomenon-like errors have been seen when the switching is occurred because of the use of the Chebyshev polynomials. In contrast, the Haar wavelet transform will make the calculation simpler, and also the discontinuity of the Haar function will be suitable for the analyses of such discontinuous behavior of the power electronics circuits. Moreover, for the field of controlling chaos [12], stabilization of the unstable periodic orbits is the main problem. However, such unstable periodic solutions as control targets are usually unknown, because it is difficult to find them by numerical calculations based on the time-marching method. If we can obtain the unstable periodic orbits by numerical calculation, it will be helpful for controlling chaos.

Therefore, in this paper, we propose the method to analyze the steady-state periodic solutions of the nonlinear circuits driven by the periodic external input by applying the appropriate boundary conditions. In the proposed method, the differential equations are transformed into the algebraic equations by using the integral and the differential operator matrices such as s in Laplace transforms. Then, by solving these algebraic equations, we can find not only the stable periodic solutions but also the unstable ones because this method is not time-marching method in which the small errors will be magnified due to instability of the unstable solutions. Moreover, the wavelet transform matrix and the operator matrices are constant matrix whose elements depend only on the wavelet level. Therefore, we can construct the wavelet-transformed differential equations by using the simple matrix operations. We will show an algorithm for the approximation of the steady-state periodic solution and the better performance for the accuracy using simple examples such as a simple boost converter and the ability to find the unstable periodic solutions with a Duffing equation.

2. Haar wavelet matrix and integral and differential operator matrices

2.1 Haar wavelet matrix

Haar functions are defined on interval $[0, 1)$ as follows,

$$h_0 = \frac{1}{\sqrt{m}}, \tag{1}$$

$$h_i = \frac{1}{\sqrt{m}} \times \begin{cases} 2^{\frac{j}{2}}, & \frac{k}{2^j} \le t < \frac{k+\frac{1}{2}}{2^j}, \\ -2^{\frac{j}{2}}, & \frac{k+\frac{1}{2}}{2^j} \le t < \frac{k+1}{2^j}, \\ 0, & \text{otherwise in } [0,1), \end{cases} \tag{2}$$

$$i = 0, 1, \cdots, m-1, m = 2^{\alpha},$$

where α is positive integer, and j and k are nonnegative integers which satisfy $i = 2^j + k$, i.e., $k = 0, 1, \cdots, 2^j - 1$ for $j = 0, 1, 2, \cdots$. Figure 1 shows the waveforms of the Haar functions for $\alpha = 2$.

We define H which is $m \times m$-dimensional Haar wavelet matrix as

$$H = [\vec{h}_0^T, \vec{h}_1^T, \cdots, \vec{h}_{m-1}^T]^T, \tag{3}$$

where $\vec{h}_i$ is $1 \times m$-dimensional Haar wavelet basis vector whose elements are the discretized expression of $h_i(t)$. For example, when $\alpha = 2$,

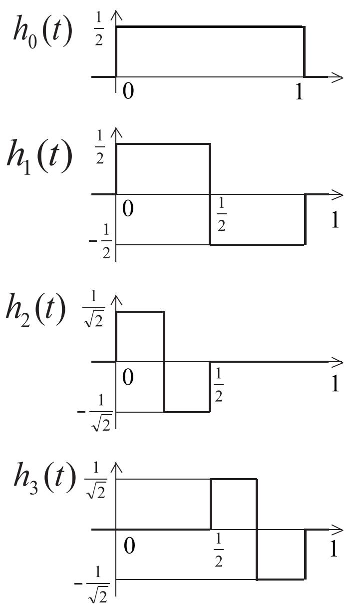


Fig. 1. Haar wavelet functions for $\alpha = 2$.

$$H = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

Note that H is an orthonormal matrix. When we consider $m \times 1$-dimensional vector $x = [x_1, x_2, \cdots, x_m]^T = [x(t_1), x(t_2), \cdots, x(t_m)]^T$ as the discretized expression of $x(t)$, Haar wavelet transform and its inverse transform are described as follows, respectively,

$$X = Hx, \tag{4}$$

$$x = H^T X (= H^{-1} X), \tag{5}$$

where X is $m \times 1$-dimensional wavelet coefficient vector.

2.2 Integral and differential operator matrices

The basic idea of the operator matrix has been firstly introduced by using Walsh function [10]. However, in logical way, the matrices introduced by block pulse function are more fundamental [10, 11]. The block pulse function is the set of m rectangular pulses which have $1/m$ width and are shifted $1/m$ each other.

The integral operator matrix of the block pulse function matrix B is defined as the following equations Eqs. (6) and (7) (see Fig. 2).

$$\int_0^t B(\tau) d\tau \equiv Q_B \cdot B(t), \tag{6}$$

$$Q_B = \frac{1}{m} \left[\frac{1}{2} I_{(m \times m)} + \sum_{i=1}^{m-1} P^i_{(m \times m)} \right], \tag{7}$$

where $I_{(m \times m)}$ is m-dimensional identity matrix, $B(t)$ is $m \times m$-dimensional matrix whose elements are the discretized expression of the block pulse functions $b_i(t),\ \ i = 0, 1, \cdots, m-1$ and

$$P^i_{(m \times m)} = \left[\begin{array}{c|c} 0 & I_{(m-i) \times (m-i)} \\ \hline 0_{(i \times i)} & 0 \end{array} \right],$$

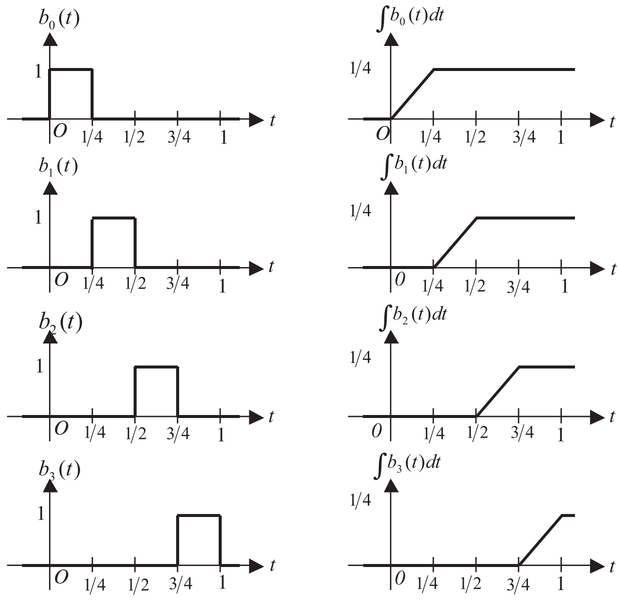


Fig. 2. Block pulse functions and their integrated functions for $\alpha = 2$.

for $i < m$,

$$P^i_{(m\times m)} = 0_{(m\times m)},$$

for $i \geq m$. And the inverse matrix $Q^{-1}_{B(m\times m)}$ is calculated as follows [11]:

$$Q^{-1}_{B(m\times m)} = 4m\left[\frac{1}{2}I_{(m\times m)} + \sum_{i=1}^{m-1}(-1)^i P^i_{(m\times m)}\right]. \tag{8}$$

By using matrix Q_B and vector x, the vector x_I which is the time series of $\int_0^t x(\tau)d\tau$ can be expressed as

$$x_I = x_0 + Q_B^T x, \tag{9}$$

where x_0 is the vector of the initial value of $x(0)$ that is $x_0 = [x(0), x(0), \cdots, x(0)]^T$. The wavelet transform of x_I is obtained from Eqs. (5) and (9)

$$Hx_I = HQ_B^T x + Hx_0 = HQ_B^T H^T X + X_0, \tag{10}$$

where $X_0 = Hx_0$. When we define the integral operator matrix $Q_H = HQ_B^T H^T$, the vector X_I which is the wavelet-domain expression of $\int_0^t x(\tau)d\tau$ can be represented as

$$X_I = Q_H X + X_0. \tag{11}$$

As the wavelet-domain expression of the derivative of $x(t)$ should be considered as inverse mapping of the integral form, the vector X_D which is the wavelet-domain expression of $\frac{dx}{dt}$ can be represented by using the operator matrix Q_H^{-1} as

$$\begin{aligned} X_D = Q_H^{-1}[X - X_0] &= H(Q_B^T)^{-1}H^T[X - X_0] \\ &= H(Q_B^{-1})^T H^T[X - X_0]. \end{aligned} \tag{12}$$

Both integral operator matrix $Q_H = HQ_B^T H^T$ and differential operator matrix $Q_H^{-1} = H(Q_B^{-1})^T H^T$ can be easily calculated from Eqs. (3), (7) and (8).

Note that the wavelet matrix H and the operator matrices Q_H and Q_H^{-1} are the constant matrices whose elements depend only on the wavelet resolution m. Therefore, simple calculations are achieved in the following sections since we can construct the wavelet-transformed differential equations by using the simple matrix operations.

2.3 Wavelet domain expression of nonlinear functions

Next, we consider the wavelet domain expression of the nonlinear function $f(x(t))$. In this paper, we assume the function has the form $f(x(t)) = g(x(t)) \cdot x(t)$. When we define m-dimensional vector $f = [f(x_1), f(x_2), \cdots, f(x_m)]^T = [g(x_1) \cdot x_1, g(x_2) \cdot x_2, \cdots, g(x_m) \cdot x_m]^T$ where $x_i = x(t_i)$, f can be rewritten as

$$f = \mathrm{diag}[g(x_1), g(x_2), \cdots, g(x_m)] \cdot x. \tag{13}$$

The Haar wavelet transform of f can be written as

$$\begin{aligned} Hf &= H\mathrm{diag}[g(x_1), g(x_2), \cdots, g(x_m)] \cdot x \\ &= H\mathrm{diag}[g(x_1), g(x_2), \cdots, g(x_m)]H^T Hx \\ &= H\mathrm{diag}[g(x_1), g(x_2), \cdots, g(x_m)]H^T X. \end{aligned} \tag{14}$$

The matrix $F \triangleq H\mathrm{diag}[g(x_1), g(x_2), \cdots, g(x_m)]H^T$ can express the nonlinearity of the system in the wavelet domain.

3. Method to find steady-state periodic solutions

Consider the following ordinary differential equation,

$$\dot{x} = f(x,t) \triangleq A(x,t)x + u(t), \tag{15}$$

where $x(t) = [x^{(1)}(t), x^{(2)}(t), \cdots, x^{(n)}(t)]^T \in R^{n\times 1}$ is an unknown state variable vector, $A(x,t) \in R^{n\times n}$ is a nonlinear time-varying parameter matrix, and $u(t) = [u^{(1)}(t), u^{(2)}(t), \cdots, u^{(n)}(t)]^T \in R^{n\times 1}$ is an external force vector. We define the initial state vector $x_0 = [x^{(1)}(0), x^{(2)}(0), \cdots, x^{(n)}(0)]^T \in R^{n\times 1}$. The system is driven by the periodic external force with period T. Assume that we can find the periodic solution $x_p(t)$ with period T, i.e. $x_p(t) = x_p(t+T)$ for all t. In order to find the steady-state periodic solutions, we should find the solution for the interval $[0,T)$ under the appropriate boundary conditions. For the wavelet expression of the differential equations, we define the discretized expression of $x^{(i)}(t)$ and $u^{(i)}(t)$ as $x^{(i)} = [x_1^{(i)}, x_2^{(i)}, \cdots, x_m^{(i)}]^T = [x^{(i)}(t_1), x^{(i)}(t_2), \cdots, x^{(i)}(t_m)]^T \in R^{m\times 1}$ and $u^{(i)} = [u_1^{(i)}, u_2^{(i)}, \cdots, u_m^{(i)}]^T = [u^{(i)}(t_1), u^{(i)}(t_2), \cdots, u^{(i)}(t_m)]^T \in R^{m\times 1}$ for $i = 1, 2, \cdots, m$, respectively. Moreover, the initial states $x_0^{(i)} = [x^{(i)}(0), x^{(i)}(0), \cdots, x^{(i)}(0)]^T \in R^{m\times 1}$ for $i = 1, 2, \cdots, n$.

Applying the relationship shown in Eq. (12), the wavelet transformed expression of Eq. (15) can be derived as

$$Q_m^{-1}[X - X_0] = A_H X + U, \tag{16}$$

where $X = [X^{(1)T}, X^{(2)T}, \cdots, X^{(n)T}]^T = [(Hx^{(1)})^T, (Hx^{(2)})^T, \cdots, (Hx^{(n)})^T]^T \in R^{mn\times 1}$ is an unknown wavelet coefficients vector, $X_0 = [X_0^{(1)T}, X_0^{(2)T}, \cdots, X_0^{(n)T}]^T = [(Hx_0^{(1)})^T, (Hx_0^{(2)})^T, \cdots, (Hx_0^{(n)})^T]^T \in R^{mn\times 1}$ and $U = [U^{(1)T}, U^{(2)T}, \cdots, U^{(n)T}]^T = [(Hu^{(1)})^T, (Hu^{(2)})^T, \cdots, (Hu^{(n)})^T]^T \in R^{mn\times 1}$. Note that $X_0^{(i)}$ are also unknown for this case. Moreover,

$$Q_m^{-1} = \begin{bmatrix} Q_H^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & Q_H^{-1} & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & Q_H^{-1} \end{bmatrix} \in R^{mn\times mn}, \tag{17}$$

and $A_H \in R^{mn\times mn}$ is the wavelet region expression of A derived from Sect. 2.3 using the matrix F.

At this moment, as both X and X_0 are unknown, we have to take into account the property $x_p(t) = x_p(t+T)$ of the periodic solutions as the boundary condition for Eq. (15). To determine the boundary condition, we define the interval as shown in Fig. 3. Due to the periodicity, the relationship $x^{(i)}(t_1) = x^{(i)}(t_m)$, i.e., $x_1^{(i)} = x_m^{(i)}$ for all $i = 1, 2, \cdots, n$ is selected as the boundary condition. From Eq. (5),

$$x_1^{(i)} = [h_{11}, h_{21}, \cdots, h_{m1}]X^{(i)}, \tag{18}$$

$$x_m^{(i)} = [h_{1m}, h_{2m}, \cdots, h_{mm}]X^{(i)}, \tag{19}$$

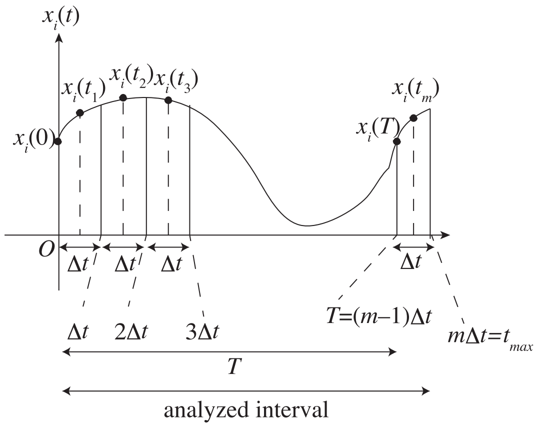


Fig. 3. Definition of the analyzed interval and the time step.

where h_{ij} is an element of Haar wavelet matrix H. Then the relationship $x_1^{(i)} = x_m^{(i)}$ is rewritten as follows,

$$[h_{11}, h_{21}, \cdots, h_{m1}]X^{(i)} = [h_{1m}, h_{2m}, \cdots, h_{mm}]X^{(i)}, \tag{20}$$

and also

$$[h_{11} - h_{1m}, h_{21} - h_{2m}, \cdots, h_{m1} - h_{mm}]X^{(i)} = 0, \tag{21}$$

Setting $h_b \triangleq [h_{11} - h_{1m}, h_{21} - h_{2m}, \cdots, h_{m1} - h_{mm}] \in R^{1\times m}$ and

$$H_b \triangleq \begin{bmatrix} h_b & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & h_b & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & h_b \end{bmatrix} \in R^{n\times mn},$$

we obtain the relationship

$$H_b X = 0. \tag{22}$$

To derive the unknown vector x_0, we consider the relationship between X and X_0. In Eq. (16), we consider the matrix $Q_H^{-1}X_0^{(i)}$ from $Q_m^{-1}X_0$ which is the term related to the initial states. From the relationship $X_0^{(i)} = Hx_0^{(i)}$,

$$Q_H^{-1}X_0^{(i)} = Q_H^{-1}Hx_0^{(i)}. \tag{23}$$

If we set a matrix $[q_{ij}] \triangleq Q_H^{-1}H \in R^{m\times m}$,

$$Q_H^{-1}Hx_{i0} = Q_H^{-1}H \begin{bmatrix} x^{(i)}(0) \\ x^{(i)}(0) \\ \vdots \\ x^{(i)}(0) \end{bmatrix} = \begin{bmatrix} q_{11} + q_{12} + \cdots + q_{1m} \\ q_{21} + q_{22} + \cdots + q_{2m} \\ \vdots \\ q_{m1} + q_{m2} + \cdots + q_{mm} \end{bmatrix} x^{(i)}(0). \tag{24}$$

When we define $q_0 \triangleq [q_{11} + q_{12} + \cdots + q_{1m}, q_{21} + q_{22} + \cdots + q_{2m}, \cdots, q_{m1} + q_{m2} + \cdots + q_{mm}]^T$ and

$$Q_0 \triangleq \begin{bmatrix} q_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & q_0 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & q_0 \end{bmatrix} \in R^{mn\times n},$$

Equation (16) is rewritten as

$$(Q_m^{-1} - A_H)X - Q_0 x_0 = U. \tag{25}$$

From Eqs. (22) and (25), we can derive $n(m+1)$-dimensional algebraic equations as follows,

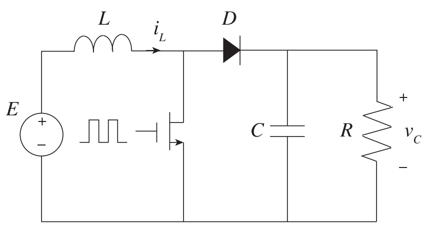


Fig. 4. Simple boost converter.

Table I. Parameter values for boost converter.

Parameter	Value
Inductance L	0.2mH
Capacitance C	0.2mF
Load resistance R	12.5Ω
Input voltage E	16V
Diode forward drop V_f	0.8V
Switching period T	100μs
On-time T_D	45μs
Switch on-resistance R_s	0.001Ω
Diode on-resistance R_D	0.001Ω

$$\left[\begin{array}{c|c} Q_m^{-1} - A_H & -Q_0 \\ \hline H_b & \mathbf{0} \end{array}\right] \left[\begin{array}{c} X \\ \hline x_0 \end{array}\right] = \left[\begin{array}{c} U \\ \hline \mathbf{0} \end{array}\right]. \tag{26}$$

In this equation, the number of the unknown variables coincides with the dimension of the equation. Therefore, we can derive all the unknown variables X and x_0 by solving it. If the system is nonlinear, Eq. (26) becomes a nonlinear algebraic equation and should be solved by the recursive methods such as Newton-Raphson method. Finally, we derive the approximated solution x of Eq. (15) from Eq. (5). In this method, the differential equations are transformed into the algebraic equations by using the integral and the differential operator matrices such as s in Laplace transforms. Then, by solving these algebraic equations, we can find not only the stable periodic solutions but also the unstable ones because this method is not a time-marching method in which the small errors will be magnified due to instability of the unstable solutions.

4. Examples

4.1 Simple boost converter

In this section, we show the simple example to confirm the effectiveness of the proposed method. The simple boost converter circuit shown in Fig. 4 is analyzed in this example. The circuit parameter is set as the same as shown in Table I. The circuit equations are written as follows,

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = \begin{bmatrix} -\dfrac{R_s(1-s(t)) + R_D s(t)}{L} & -\dfrac{s(t)}{L} \\ \dfrac{s(t)}{C} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \dfrac{E - s(t)V_f}{L} \\ 0 \end{bmatrix}, \tag{27}$$

where

$$s(t) = \begin{cases} 0, & \text{for } 0 \le t \le T_D \\ 1, & \text{for } T_D \le t \le T \\ s(t-T) & \text{for all } t > T. \end{cases} \tag{28}$$

Equation (27) can be transformed in wavelet-domain as follows,

$$\begin{bmatrix} Q_H^{-1} & \mathbf{0} \\ \mathbf{0} & Q_H^{-1} \end{bmatrix} \begin{bmatrix} I_L - I_{L0} \\ V_C - V_{C0} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & -\dfrac{1}{RC} I_{(m\times m)} \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} U \\ \mathbf{0} \end{bmatrix}, \tag{29}$$

where

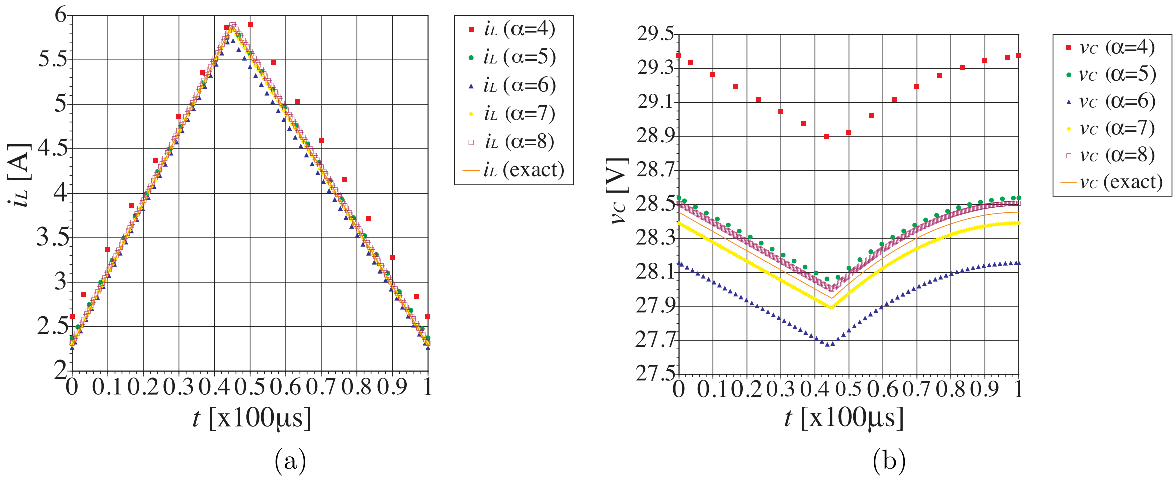


Fig. 5. Numerical results of i_L and v_c for the proposed method for $\alpha = 4, 5, 6, 7, 8$.

Table II. Comparison of MREs for approximation in boost converter.

α	MRE for i_L ([9])	MRE for v_C ([9])	MRE for i_L (proposed)	MRE for v_C (proposed)
4	0.025789	0.025714	0.078896	0.032298
5	0.025745	0.025704	0.011233	0.003098
6	0.025736	0.025703	0.019376	0.010234
7	0.025734	0.025702	0.003816	0.002181
8	0.031353	0.028476	0.004065	0.001844

$$F_{11} = H\mathrm{diag}[R_s, \cdots, R_s, R_D, \cdots, R_D]H^T, \tag{30}$$

$$F_{12} = H\mathrm{diag}\left[0, \cdots, 0, -\frac{1}{L}, \cdots, -\frac{1}{L}\right]H^T, \tag{31}$$

$$F_{21} = H\mathrm{diag}\left[0, \cdots, 0, \frac{1}{C}, \cdots, \frac{1}{C}\right]H^T. \tag{32}$$

An example of the calculated results for the proposed method are shown in Fig. 5. From these figures, we can see good approximation is achieved compared with the exact solutions. To evaluate the accuracy of the proposed method, we calculate mean relative error (MRE) given by

$$\mathrm{MRE} = \frac{1}{m+1}\sum_{j=0}^{m}\left|\frac{x_j^{(i)} - \hat{x}^{(i)}(t_j)}{\hat{x}^{(i)}(t_j)}\right|, \tag{33}$$

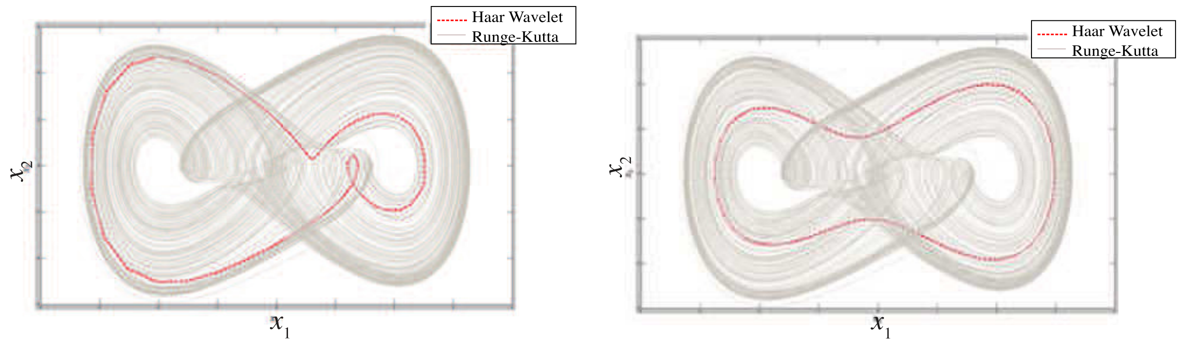
where $\hat{x}$ means the exact solutions. Table II shows the MRE for i_L and v_C compare with the errors shown in [9]. From this results, the proposed method can achieve the better approximation than the method shown in [9] except the case of $\alpha = 4$, which is low-resolution case. Hence, we can construct the simple and accurate method to calculate the steady-state periodic solution by using Haar wavelet transform. It is considered that the proposed method can play important roles to analyze the behavior of the various kinds of circuits such as the power electronic circuits and hybrid dynamical systems.

4.2 Duffing equation

In this paper, as an example, we use Duffing equation as shown in Eq. (34), which is one of the best-known continuous-time chaotic systems,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\varepsilon x_2 - \beta x_1 - b^2 x_1^3 + u\cos\omega t. \end{cases} \tag{34}$$

We can apply the Haar wavelet transform to the state equations as follows.



(a) (b)

Fig. 6. Examples of the periodic solutions for parameter set $(\varepsilon, \beta, b, u, \omega)$. (a) $(0.25, 0, 1, 8.2, 1)$, (b) $(0.25, 0, 1, 9, 1)$.

$$\begin{bmatrix} Q_H^{-1} & \mathbf{0} \\ \mathbf{0} & Q_H^{-1} \end{bmatrix} \begin{bmatrix} X_1 - X_{10} \\ X_2 - X_{20} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & I_{(m\times m)} \\ F & -\varepsilon I_{(m\times m)} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ U \end{bmatrix}, \tag{35}$$

where $F = H\mathrm{diag}[-\beta - b^2x_1^2, -\beta - b^2x_2^2, \cdots, -\beta - b^2x_m^2]H^T$. By applying Eq. (35) to Eq. (25) and solving this algebraic equation, we can find the periodic solution in the system.

Figure 6 shows some examples of the periodic solution with the parameters with which the chaotic oscillations are stably excited. In these results, thin solid curves describe the chaotic attractors calculated by 4-th order Runge-Kutta method, and red curves show the periodic solutions obtained by the proposed Haar wavelet method. From these results, unstable periodic solutions in chaotic parameter regions can be numerically obtained by proposed methods. From these results, the proposed method seems to have the ability to find various kinds of periodic solutions embedded in chaotic dynamical systems. However, we cannot verify those solutions are correct because they are unstable and cannot be analyzed by the time-marching methods such as Runge-Kutta method. The verifications of the solutions are our future problems.

5. Conclusions

In this paper, we have proposed the method to analyze the steady-state periodic solutions of the nonlinear circuits driven by the periodic external input such as a simple boost converter and Duffing equation by applying the boundary conditions that $x_p(t) = x_p(t+T)$. From the calculation results, we have obtained the accurate and appropriate solutions by the proposed method. The Haar wavelets make the algorithm simpler and the better accuracy has been achieved compared with the methods previously shown. Therefore, it is considered that the proposed method can play important roles to analyze the power electronic circuits and hybrid dynamical systems. In addition, it will be a helpful tool for finding the unstable periodic orbits in the field of chaos control. The application of the proposed method to the autonomous systems seems to be the future works.

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