Robust Control and Sensitivity Analysis of 2-link Manipulator under Torque Constraints

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Robust Control and Sensitivity Analysis of 2-link Manipulator under Torque Constraints

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This paper is concerned with the control strategy of a robot manipulator system. In designing the control strategy, there will be always some difference between the actual behavior of the physical system and that predicted by the mathematical model. This difference brings the unstable behavior to the system. Then, it is important to assure a degree of "robustness" or immunity to uncertainty or changes in the system parameters.

First, in order to implement the robust control, the model reference adaptive control theory is applied to the linearlized manipulator system. Next, through the simulation studies, it is clarified that the stability of the system is not lost by parameters change but lost under torque constraints. Finally, the sensitivity analysis is applied to the investigation of the influence of parameter difference from nominal values on the dynamics of a robot manipulator. Through this analysis, we can find which parameters really influence to the manipulator control and which do not.

1. Introduction

When obtaining the mathematical model of the dynamics with respect to a manipulator and constructing an adaptive control system based on this model, there arise the modeling errors since the system parameters can not be accurately estimated. Even if there

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exist more or less errors, it is extremely important to maintain the stable control performance (robustness) from the practical viewpoint. This paper develops some problems in the case where the robust control strategy is applied to a 2-link manipulator.

The manipulator is a nonlinear dynamical system with the interactions among various joints. Applying the non-interaction control theory [1),2),3)] to a 2-link manipulator, individual outputs may be non-interacted and transformed into linear systems. Then, the optimal control strategy [3),4),5)] can be applied to this linearlized system, which is one of robust control theories. In order to confirm the effects of robust controller, simulation studies are performed which the manipulator follows a prescribed path. Especially, under torque constraints between each joints, the dynamics of the manipulator has been analized.

Furthermore, when system parameters of the manipulator which has been linearlized by the non-interacting control theory change from nominal values, the rates of the effect for the system dynamics are investigated by the sensitivity analysis.[6)]

2. Mathematical Model of the Manipulator

Figure 1 shows a 2-link manipulator. As generalized joint variables θ_i (i=1, 2), we denote the relative angular position between links at i-th joint. The masses and lengths of both links 1 and 2 are relatively m_1, m_2, ℓ_1 and ℓ_2. The manipulator mathematical model can be written in the form; [1),2)]

$$M\ddot{\Theta} + D\dot{\Theta} + G = \tau \qquad (1)$$

where τ is the vector of torques acting on the joints, M the matrix of inertia, D the matrix of Coliolis and centrifugal forces and G the vector of gravitational torques.

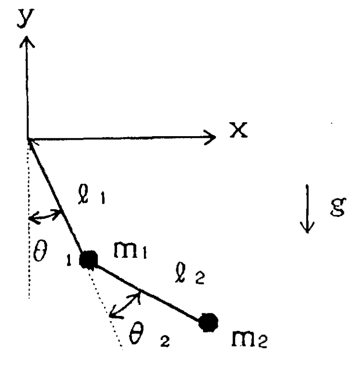


Fig.1 2-link manipulator

For the mathematical model of Eq.(1), the estimation of unknown parameters can be performed on the basis of measured values of Θ and $\dot{\Theta}$. From above results, the dynamical equations can be, in practice, represented by

$$M\ddot{\theta} + \tilde{h}_o = \tau_o \qquad (2)$$

$$\tilde{h}_o = D_o\theta + G_o \qquad (3)$$

Here, it is assumed that the driving torque τ of the manipulator is the input which can be directly regulated. Let be $x=[\theta,\dot{\theta}]^T$ as the state vector and $y=\theta$ as the observation vector. Applying the non-interaction control theory [1),2),3)] to Eqs.(2) and (3), the following control input τ_o is obtained.

$$\tau_o = M_o \mathrm{col}[\sum_{i=1}^{2}(\lambda u_i - \alpha_1\theta_i - \alpha_o\theta_i)] + D_o\dot{\theta} + G_o \qquad (4)$$

(col[·] denotes the column vector)

Substituting τ_o of Eq.(4) into τ of Eq.(1), we have

$$\ddot{\theta} = M^{-1}M_o\,\mathrm{col}[\sum_{i=1}^{2}(\lambda u_i - \alpha_1\theta_i - \alpha_o\theta_i)] + M^{-1}\Delta D_o\dot{\theta}$$

$$+ M^{-1}\Delta G, \qquad (5)$$

where $\Delta D = D_o - D$, $\Delta G = G_o - G$.

If both dynamical equations of Eqs.(1) and (5) are perfectly consistent, it becomes that $M^{-1}M_o = I$, $\Delta D = 0$ and $\Delta G = 0$ in Eq.(5). Letting $x=[y_1, y_2]^T$, the following non-interaction system may be obtained.

$$\dot{x} = Ax + Bu \qquad (6)$$

$$y = Cx \qquad (7)$$

where

$$A = \begin{bmatrix} 0 & \mathbf{I} \\ -\alpha_0 & -\alpha_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \lambda \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix}. \qquad (8)$$

3. Design of the Adaptive Controller

Applying the model reference adaptive control systems (MRACS) techniques[3),4),5)] to the manipulator dynamical equation (6), the optimal regulator is constructed as follows.

Assume that $x_d = [y_{d1}, y_{d2}]^T$ represents a desired trajectory in joint space that the manipulator is to track. Then, the dynamics may be written by

$$\dot{x}_d = A_d x_d + B_d u_d \tag{9}$$

$$y_d = C_d x_d, \tag{10}$$

where

$$A_d = \begin{bmatrix} 0 & \mathbb{I} \\ -a_o & -a_1 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ a_o \end{bmatrix}, \quad C_d = \begin{bmatrix} 0 \\ \mathbb{I} \end{bmatrix}. \tag{11}$$

Now, using Eqs.(6) and (9), we shall consider the optimal control of the manipulator, based on the adaptive following servo control(AFSC) technique. Let's define $e \triangleq y_d - y$ as the error between the model state y and the desired path y_d. Differentiating Eqs.(6) and (9), we have,

$$\dot{x}^* = A^* x^* + B^* \dot{u} \tag{12}$$

$$e^* = C^* x^* \tag{13}$$

where

$$x^* = [\ \dot{x} \quad e \quad x_d\ ]^T$$

$$A^* = \begin{bmatrix} A & 0 & 0 \\ -C & 0 & C_d \\ 0 & 0 & A_d \end{bmatrix}, \quad B^* = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \tag{14}$$

$$C^* = [\ 0 \quad \mathbb{I} \quad 0\ ]$$

For Eq.(12), the following quadratic cost function[3),4)] is used:

$$J(u) = \int_0^\infty (e^T Q e + \dot{u}^T R \dot{u})\, dt \tag{15}$$

where Q and R are positive definite 2×2 symmetric matrix.

The control u which minimize $J(u)$ to Eq.(12) is called to be an optimal input. The control input is formulated as;[3),7)]

$$\dot{u} = K x^*, \tag{16}$$

where the optimal gain matrix K is given by

$$K = [\ K_b \quad K_i \quad K_f\ ] = -R B^{*T} P, \tag{17}$$

where P is the unique positive definite symmetric solution to the following Riccati equation;

$$A^{*T} P + P A^* + Q + P B^* R^{-1} B^{*T} P = 0. \tag{18}$$

Using Eq.(17), the integral of Eq.(16) under the initial condition $x(0) = x_d(0) = 0$ becomes as

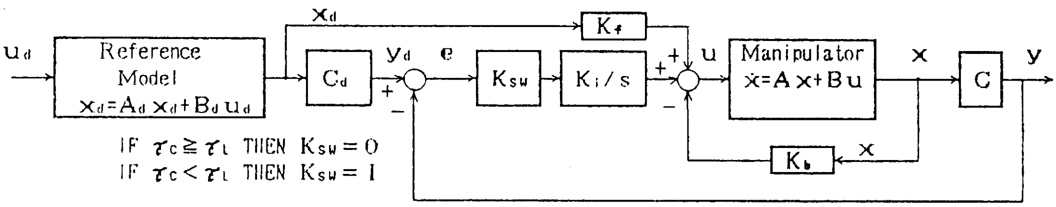


Fig.2 Construction of MRACS (Optimal Control System)

$$u = K_b x + K_i \int_0^t e\,dt + K_f x_d \ . \tag{19}$$

From described above, the construction of the model following servo system is shown by the block diagram in Fig.2. Here, K_{sw} will be explained later. In order to determine the values of K, the desired responses were examined by changing values of both Q and R in Eq. (15) on trial and error. At result, we had $Q=1\times10^7$ and R=1. Also, the optimal gain K was obtained as K = [433 28.6 3160 343 15.4]. Using K obtained above, simulation experiments are carried out.

4. Simulation Experiments

4.1 Manipulator Dynamics due to the Addition of the Load

In the case where the load is added to the manipulator, parameters of the dynamical equation (1) may change and become disagreed with ones of Eq.(2). Accordingly, since the nonlinear term occures for the linearlized equation (6) as the error term, the manipulator dynamical equation becomes nonlinear one as Eq.(5). In order to achieve the perfect non-interaction control, it is necessary to recalculate the control scheme by identifying Eq.(2) again. However, it is in fact difficult to estimate system parameters because of variations in the load. Then, when the nonlinear term occures for the linearlized equation, we will examine, through simulation experiments, whether the optimal regulator operates in the stable state for any extent of the load.

For the optimal controlled manipulator, we consider that the load is added to the endpoint of link 2. In this case, supposed that there is no load in Eq.(2), there arise errors between the system equation (1) and the model one (2). Thus, when there exist errors, we shall examine the effect of the optimal control strategy for the manipulator. Here, initial states of the manipulator are $\theta_1 = \theta_2 = 0$

[rad]. As the desired values, the different ramp inputs are given for both link 1 and link 2. Also, loads added to the manipulator are m_p=1[kg] and 7[kg], respectively. The simulation experiments are performed using the model reference adaptive control technique as shown in Fig.2.

Figures 3, 4 and 5 show responses of the manipulator obtained through simulation studies, in which Fig.3 is the case of no optimal control and Figs.4 and 5 ones of optimal control. From these results, the manipulator follows the desired trajectories so far as the load of m_p=7[kg]. At this time, the mass of only link 2 is approximately m_{c2}=5[kg] and $m_2=m_{c2}+m_p$=13[kg] with load 7[kg]. In spite of 2.6 times of the weight compared with the case of no load, the favorable responses can be realized. From this fact, even if there are some differences of parameters between the practical model (1) and the linearlized one (2), it has been confirmed that it might be possible to keep the robust control of the manipulator.

4.2 Manipulator Dynamics under Torque Constraints

In the previous section, it was supposed that the motor to regulate the manipulator generated the torque proportional to the input signal. However, from the practical viewpoint, the manipulator is subject to bounds on the available torque at each joints. In this section, the effect of the torque constraints is examined through simulation studies.

As shown in Fig.4, it was obtained that the maximum torque to regulate the manipulator is, with no load i.e. m_p=0[kg], 85.3[N·m]. From this fact, it was assummed that the maximum torque was limited to τ_L = 80.0 [N·m]. In this case, the block diagram of the manipulater is shown in Fig.6. The responses and the torque variations, concerning with both the link 1a and 2a, are shown in Fig.7(a) and (b). From the dynamics of both links 1a and 2a in Fig.7, it can be understood that there occures vibration on the response of the manipulater, even if the maximum torque is slightly limited. As the cause that the system becomes unstable, when the torque τ is limited, it may be considered to be windup phenomenon[9] of the integrator. This phenomenon is explained as follows. When the torque τ takes the limit, the torque τ becomes constant and doesn't change. On the otherhand, the error e is added and stored to the integrator K_i/s as an input. Thus, when the error e takes the inverse sign, the torque τ can not change at once because of the extra quantity storred in the integrator K_i/s. Accordingly, when the torque τ

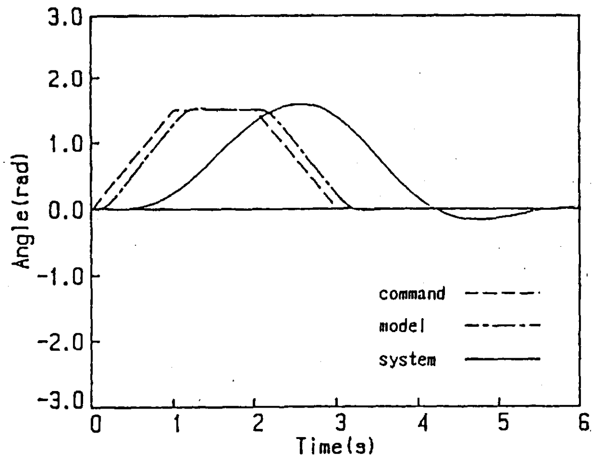


Fig.3 Response of manipulator (No optimal control)

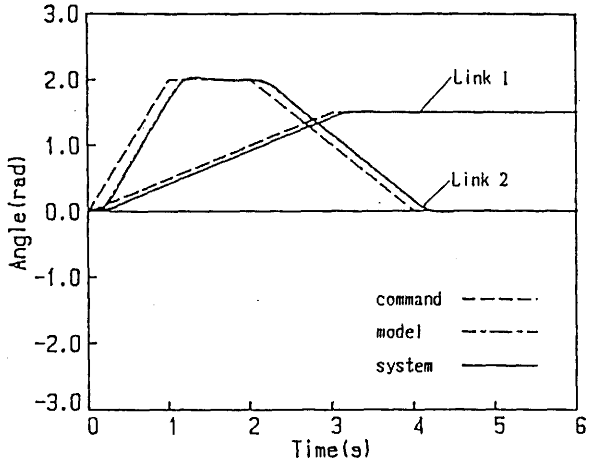


Fig.4 Response of manipulator-1 (m_p=1[kg])

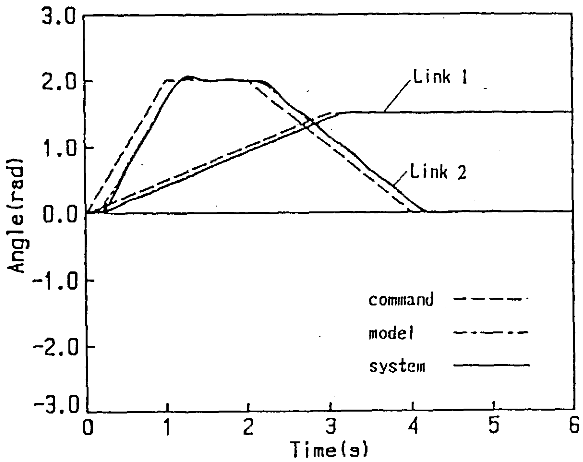


Fig.5 Response of manipulator-2 (m_p=7[kg])

takes the upper or lower limit, the amplitude of the controlled volume y increases and causes the oscillation in the worst case. Then, such phenomenon is called to be windup phenomenon of the integrator.

As the simplest method to suppress the windup phenomenon, there is a method to hold the content of the integrator while the torque τ takes the limit. This strategy is, as shown in Fig.2, to set that $K_{sw} = 0$ if $\tau_c \geq \tau_L$ and $K_{sw} = 1$ if $\tau_c < \tau_L$. In order to confirm the effect of this method, the simulation experiment has been performed under same conditions as Fig.6. Figure 7 shows both (a) responses of link 1 and link 2 and (b) the torques variations, in which link 1a and link 2a are the case of oscillation under the torque constraints and link 2a and link 2b the case with the windup countermeasure. From results in Fig.7 for the optimal control system including the windup countermeasure, the responses of the manipulator can be improved to some degree.

From simulation experiments described above, it has been clarified that the robustness of the optimal control system may be lost under the torque constraints of the electric-motor.

5. Sensitivity Analysis

5.1 Sensitivity Model

For the sensitivity analysis, the parameters uded here are listed in Table 1.

Table 1 Numerical values of link parameters

Parameter	values	Parameter	values
$p_1 = m_{c1}$	5.0[kg]	$p_5 = \alpha_0$	100.0
$p_2 = m_{c2}$	5.0[kg]	$p_6 = \alpha_1$	14.14
$p_3 = \ell_{c1}$	0.5[kg]	$p_7 = \lambda$	100.0
$p_4 = \ell_{c2}$	0.5[kg]		

Now, let be $x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$ and $x_4 = \dot{\theta}_2$. Also, denote that

$$x = [\, x_1, x_2, x_3, x_4 \,]^T,$$
$$u = [\, u_1, u_2 \,]^T,$$
$$p = [\, p_1, p_2, \cdots\cdots, p_7 \,]^T$$

and $A = m_{11}m_{22} - m_{21}m_{12}$.

Then, using parameters as shown in Table 1, a mathematical model of Eq.(5) can be represented by the following state equation,

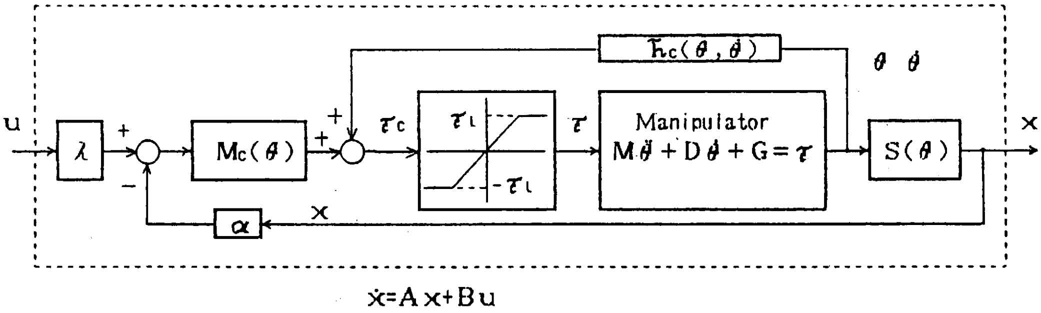
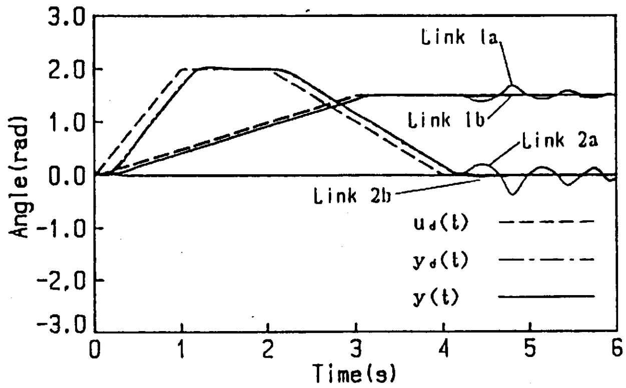
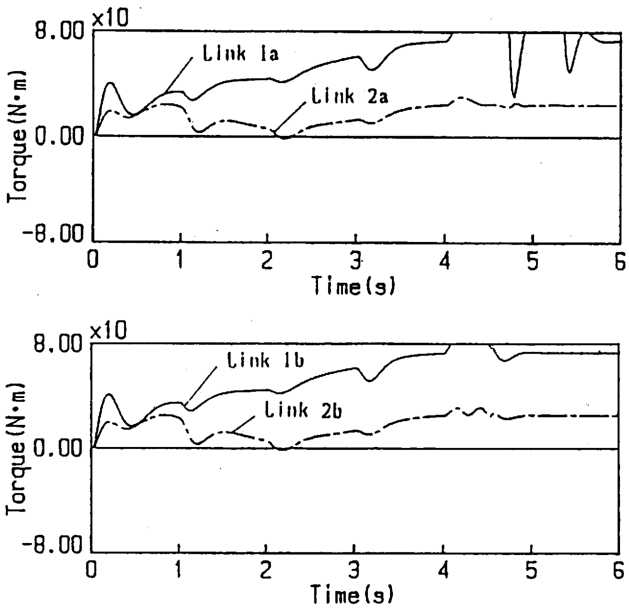


Fig.6 Block diagram of non-interaction control system



(a) Responses of link 1 and link 2



(b) Torques developed at manipulator joints

Fig.7 Responses of manipulator-3
(Improvement of windup phenomenon)

$$\dot{x} = f(\, x,\ u,\ p\,)$$

$$= \begin{bmatrix} x_2 \\ [-a_{11}p_5x_1 + (b_{11}-a_{11}p_6)x_2 - a_{12}p_5x_3 + (b_{12}-a_{12}p_6)x_4 \\ \quad + a_{11}p_7u_1 + a_{12}p_7u_2 + c_1]/A \\ x_4 \\ [-a_{21}p_5x_1 + (b_{21}-a_{21}p_6)x_2 - a_{22}p_5x_3 + (b_{22}-a_{22}p_6)x_4 \\ \quad + a_{21}p_7u_1 + a_{22}p_7u_2 + c_2]/A \end{bmatrix}. \quad (20)$$

By the way, the solution of Eq.(20) is, in general, given by

$$x_n(t) = \phi(\, t,\ p_n\,), \qquad (21)$$

where the suffix n represents the nominal value. From Eq.(21), the vector sensitivity function Z is defined as,[6)]

$$Z^j = \left(\frac{\partial\, x}{\partial\, p_j} \right)_n, \qquad j = 1, 2, \cdots\cdots, 7. \qquad (22)$$

Assuming that u is independent on p and differentiating Eq.(20) partially with respect to p, we obtain the sensitivity equations in the form,

$$\dot{Z}^j = \left(\frac{\partial f}{\partial x} \right)_n Z^j + \left(\frac{\partial f}{\partial p_j} \right)_n, \quad j = 1, 2, \cdots\cdots, 7 \qquad (23)$$

where $(\, \partial f / \partial x\,)_n$ is the Jacobian matrix evaluated on the nominal solution.

The initial state for Eq.(23) is Z_0^j. Also, the initial one of Eq.(20) is assumed to be $x_0(0)=\phi(0,p_n)$. Letting $x_0 = 0$ at $t_0=0$, the initial condition of the sensitivity function is $Z_0^j = 0$.

In addition, we shall consider the differential variation δx of state variables due to the parameter variation,

$$\delta p = p - p_n. \qquad (24)$$

Then, the variation of state variables becomes,

$$\delta x(t) = \phi(t, p_n) - \phi(t, p). \qquad (25)$$

Using Taylor's expansion theorem, the first approximation of the variation δx may be written

$$\delta x(t) \cong \left(\frac{\partial x}{\partial p} \right)_n \delta p\ . \qquad (26)$$

From Eq.(26), we can find the differential variation δx of state variables due to the parameter variation.

The magnitude of the output $\delta\Theta = [\,\delta\theta_1,\ \delta\theta_2\,]^T$, which fluctuates due to each parameters variation δp_j ($j = 1,2,\cdots\cdots,7$), can be obtained, using Eqs.(22) and (26), as

$$\delta\Theta = \sum_{j=1}^{7} z^j\,\delta p_j\,. \qquad (27)$$

5.2 Simulation Experiments

Through simulation experiments, the effect has been examined that the slight variation of each parameters gives on the response of the manipulator. The sensitivity analyses were performed by selecting 5% variation of any parameters from their nominal value. For parameters m_{c1} and m_{c2} of the manipulator, the responses of output angle $\delta\Theta$ are shown in Figs.8 and 9, which were obtained from the computation of Eq.(27). In these figures, the response $\delta\theta_1$ of link 1 in Fig.9, due to the parameter m_{c2}, fluctuates more than one in Fig.8. Since the maximum value is $\delta\theta_1$=7.13×10^{-2}[rad], it becomes $|\delta\theta_1/\theta_1|$=0.0713 where the final moving angle θ_1=1.0[rad].

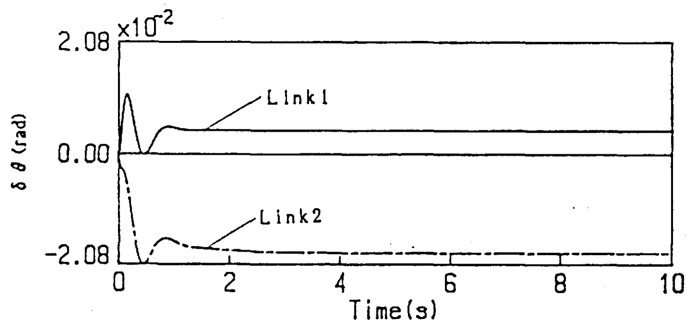


Fig.8 Response of $\delta\Theta$ due to the variation of m_{c1}

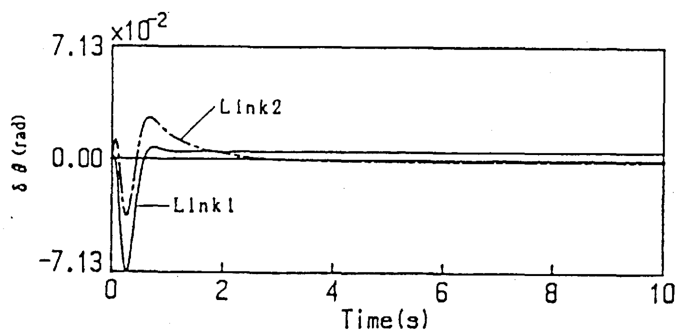


Fig.9 Response of $\delta\Theta$ due to the variation of m_{c2}

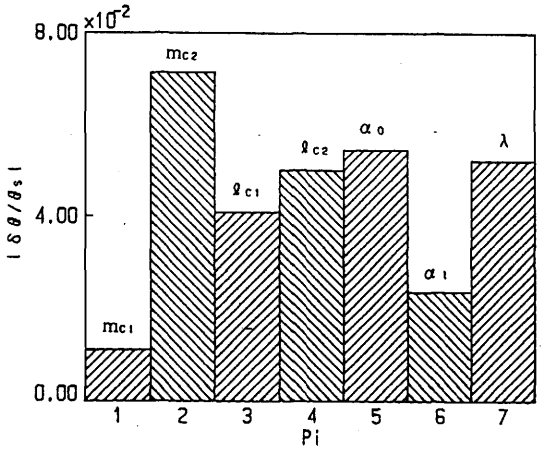


Fig.10 Comparison of parameters influence on the output θ_1

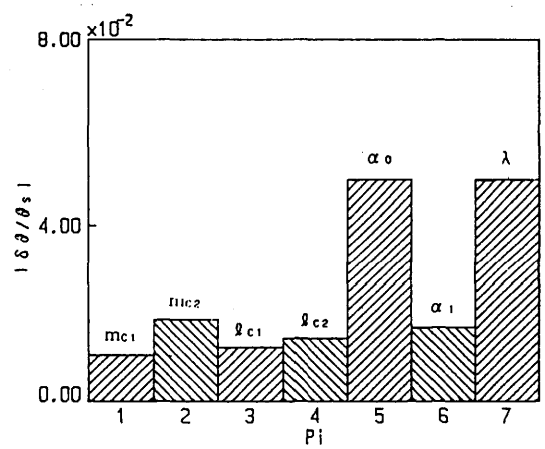


Fig.11 Comparison of oarameters influence on the output θ_2

Accordingly, it has the error of about 7.1%. Similarly, we examine the rate of the effect that other parameters variation influences on the response of the manipulator. Letting Θ_s be the steady state value, the maximum values of $|\delta\Theta/\Theta_s|$ are given by

$$\left|\frac{\delta\Theta}{\Theta_s}\right| = \frac{|z^j|_{max}|\delta p_j|}{|\Theta_s|}, \tag{28}$$

where $|\delta p_j| = 0.05\,p_j$. The results of the computation for Eq.(28) are, as histograms, shown in Figs.10 and 11, in which one column represents a maximum variation in the nominal step response of $\delta\Theta$ scaled by the steady state value Θ_s. Figure 10 shows each parameters influence on the output response θ_1 of link 1, and Fig.11 on the response θ_2 of link 2.

From both figures, we consider the effect of variation on m_{c1}, m_{c2}, ℓ_{c1} and ℓ_{c2} to the manipulator. Concerning with link 1 in Fig.10 and link 2 in Fig.11, it may be noted that the effects of m_{c2} and ℓ_{c2} are larger than ones of m_{c1} and ℓ_{c1}. Also, it can be understood that, when each parameters fluctuate, the rate of effect for the output angle θ is stronger on link 2 than on link 1.

Next, we notice the effect of each parameters for each links. It can be seen that, in link 1 of Fig.10, a parameter m_{c2} is the most effective on the dynamics. The rate of the effect that the variation of nonlinear feedback parameters α_0, α_1, λ influences on the response of the manipulator is almost the same degree for both links 1 and 2. For that reason, with respect to link 2 in Fig.11, as the effect of system parameters m_{c1}, m_{c2}, ℓ_{c1} and ℓ_{c2} is small, the influences of parameters α_0, α_1 and λ become relatively large. Especially, among 7 parameters of link 2, the most effective parameter is α_0. The effect of α_0 is about 2.5 times, comparing with m_{c2} which is the most effective among parameters m_{c1}, m_{c2}, ℓ_{c1} and ℓ_{c2}.

6. Conclusions

In this research, we obtain the followings.

(1) In order to ascertain the varidity of the optimal control strategy for a manipulator, in which the optimal gain K was determined under the assumption of no load, the simulation experiments had been performed under the condition that the load was added at the endpoint of the manipulator. Consequently, even if the load was about 1.5 times for the link weight, the stable responses were obtained. For that, it has been clarified for this system to construct the robust optimal control system.

(2) Under the torque constraints, which means the case where the torque needed to control the manipulator exceeds the maximum limit of the joint torque, the system generates oscillation and unstable state which is called windup phenomenon. In this case, it was confirmed that the response was improved to a certain degree by a windup countermeasure. However, it was clarified that the robustness of the optimal control system was generally lost under torque constraints of the joint motor.

(3) Through the sensitivity analysis, it has been examined that the parameters variation influences on the response of the manipu-

lator to what extent. At result, it was verified that the system parameter of link 2 influenced on the response more than ones of link 1 and the response of link 1 was sensitive to the parameters variation more than one of link 2.

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