Analysis Method of Periodic Solution using Haar Wavelet Transform for Autonomous Nonlinear Circuits

Analysis Method of Periodic Solution using Haar Wavelet Transform for Autonomous Nonlinear Circuits

Tatsuya Nakabayashi, Masataka Mochizuki, and Seiichiro Moro
Department of Electrical and Electronics Engineering, University of Fukui
3-9-1 Bunkyo, Fukui, 910-8507 Japan
E-mail: tnfb1020@gmail.com, mochizuki@ppc8100.fuee.u-fukui.ac.jp, moro@u-fukui.ac.jp

***Abstract*— Recently, much attemtion has been paid to the methods for circuit analysis using wavelet transform. In particular, we have proposed the analysis methods using Haar wavelet transform. Haar wavelet can be easily treated, be adapted to time varying and nonlinear circuits and we can be easily derived differential and integral operator matrices by using block pulse functions. Furthermore, it can analyze a region near singular points more precisely. Therefore, we consider that it is suitable to analyze nonlinear time-varying circuits. In this paper, we propose the method to analyze the numerical solution of the periodic solution of the autonomous nonlinear circuit.**

Keywords—Haar Wavelet transform, periodic solution, autonomous circuits

I. INTRODUCTION

Recently, much attention has been paid to the methods for circuit analysis using wavelet transform [1]-[7]. We proposed the steady-state periodic solution analysis methods using Haar wavelet transform [2], [3], [5], [7]. Haar wavelet can be easily treated, be adapted to time varying and nonlinear circuits and we can be easily derived differential and integral operator matrices by using block pulse functions. These properties are based on the methods using Walsh transforms [8], [9]. Furthermore, from the orthogonality and the locality of Haar wavelet, it can analyze the region near singular points where the solution is steep more precisely. Then, we proposed the method which can analyze the region near singular points with adaptive resolutions [3].

Moreover, the analysis methods for power electronics circuits using wavelet transform were proposed by Tam et al [4]. If we calculate steady-state waveforms of power electronic circuits using the successive integral methods such as the Runge-Kutta methods, the calculation cost is wasted due to the calculation of the long term transient response with sufficiently small step size to approximate the discontinuous dynamics caused by switches. To overcome such disadvantage of the successive integral method, in [4], the Chebyshev polynomials are used as the wavelet basis functions, and the periodic solutions of periodically driven power electronic circuits are calculated. However, it is considered that the calculation should be complicated and the Gibbs-phenomenon-like errors have been seen when the switching is occurred because of the use of the Chebyshev polynomials. Therefore, we proposed the analysis methods using Haar wavelet transform in the non-autonomous nonlinear circuits for steady-state periodic solution [7]. This method can derive the steady-state solution of the circuit because the period of the solution is determined by input excitations. And, we do not need to calculate the transient state of the circuit like the successive integral methods such as the Runge-Kutta methods. Therefore, these can shorten calculation time very much.

However, these proposed methods [4], [7] cannot analyze the autonomous nonlinear circuits because the period of the solution is unknown. Therefore, using optimization method, we use the method to obtain the period of the circuit by deriving the minimum value of norm of the state equations [6]. Therefore, we consider that we can derive the periodic solution by using the method. In this paper, we propose the method to analyze the numerical solution of the periodic solution of the autonomous nonlinear circuit by using Haar wavelet transform and optimization method. And, we show calculation result of this method when a van der Pol oscillator is an example circuit.

II. HAAR WAVELET MATRIX

Haar functions are defined on interval $[0,1)$ as follows,

$$h_{\mathbf{0}} = \frac{\mathbf{1}}{\sqrt{\boldsymbol{m}}} \tag{1}$$

$$h_i = \frac{1}{\sqrt{m}} \times \begin{cases} 2^{\frac{j}{2}}, & \frac{k-1}{2^j} \le t < \frac{k-\frac{1}{2}}{2^j}, \\ -2^{\frac{j}{2}} & \frac{k-\frac{1}{2}}{2^j} \le t < \frac{k}{2^j}, \\ 0 & otherwise\ in\ [0,1) \end{cases} \tag{2}$$

$$i = 0,1,\cdots,m-1, m = 2^a$$

where a is positive integer, and j and k are nonnegative integer which satisfy $\mathrm{i} = 2^j + k$, i.e., $k = 0,1,\cdots,2^j - 1$ $(j = 0,1,2,\cdots)$. Figure 1 shows the waveforms of the Haar functions for $a = 2$.

$\boldsymbol{H}$ is $m \times m$-dimensional Haar wavelet matrix defined as

$$\boldsymbol{H} = \begin{bmatrix} \overrightarrow{h_0} \\ \overrightarrow{h_1} \\ \vdots \\ \overrightarrow{h_{m-1}} \end{bmatrix} \triangleq \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1m} \\ h_{21} & h_{22} & \cdots & h_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mm} \end{bmatrix} \tag{3}$$

where $\overrightarrow{h_l}$ is $1 \times m$-dimentional Haar wavelet basis vector whose elements are the discretized expression of $h_i(\mathrm{t})$. Note that $\boldsymbol{H}$ is an orthonormal matrix. $\vec{y}$ is $m \times 1$-dimentional vector whose elements are the discretized expression of $y(t)$ which is the function that has square integrability on interval $[0,1)$. $\vec{c}$ is $m \times 1$-dimentional coefficient vector. Using these vectors and matrix, Haar wavelet transform and inverse Haar wavelet transform are described as follows.

$$\vec{c} = \boldsymbol{H}\vec{y} \tag{4}$$

$$\vec{y} = \boldsymbol{H}^T\vec{c}(= \boldsymbol{H}^{-1}\vec{c}) \tag{5}$$

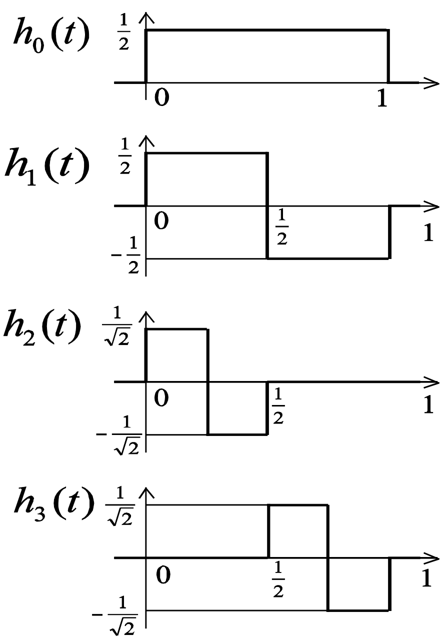

Fig. 1 : Haar wavelet function for $a = 2$.

III. Integral and Differential Operator Matrices using Haar Wavelet

Historically, the basic idea of operator matrix has been introduced by using Walsh functions [9]. However, in logical way, the matrices introduced by block pulse function are more fundamental. The block pulse function is the set of m rectangular pulses on interval [0,1) which have $1/m$ pulse width and are shifted $1/m$ each other.

The integral operator matrix of the block pulse function matrix $\boldsymbol{B}$ is defined as the following equation [4]-[5].

$$\int_0^t \boldsymbol{B}(\boldsymbol{\tau})d\boldsymbol{\tau} \equiv \boldsymbol{Q_B} \cdot \boldsymbol{B}(\boldsymbol{t}) \tag{6}$$

$$\boldsymbol{Q}_{\boldsymbol{B}(\boldsymbol{m}\times\boldsymbol{m})} = \frac{\boldsymbol{1}}{\boldsymbol{m}}\left[\frac{1}{2}\boldsymbol{I}_{(\boldsymbol{m}\times\boldsymbol{m})} + \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\infty}\boldsymbol{P}^{\boldsymbol{i}}_{(\boldsymbol{m}\times\boldsymbol{m})}\right] \tag{7}$$

Where $\boldsymbol{B}$(t) is $\mathrm{m} \times \mathrm{m}$ -dimensional matrix whose elements are the discretized expression of the block pulse function $b_i(\mathrm{t})$, $i = 0,1,2,\cdots,m-1$ and

$$\boldsymbol{P}^{\boldsymbol{i}}_{(m\times m)} = \left[\begin{array}{c|c} \boldsymbol{0} & \boldsymbol{I}_{(\boldsymbol{m}-\boldsymbol{i})\times(\boldsymbol{m}-\boldsymbol{i})} \\ \hline \boldsymbol{0}_{(\boldsymbol{i}\times\boldsymbol{i})} & \boldsymbol{0} \end{array}\right]$$

for $i < m$,

$$\boldsymbol{P}^{i}_{(m\times m)} = \boldsymbol{0}_{(m\times m)}$$

for $i \geq m$. And for $i < m$, the inverse matrix $\boldsymbol{Q}^{-1}_{\boldsymbol{B}(m\times m)}$ is calculated as follows [5].

$$\boldsymbol{Q}^{-1}_{\boldsymbol{B}(m\times m)} = 4m\left[\frac{1}{2}\boldsymbol{I}_{(m\times m)} + \sum_{i=1}^{m-1}(-1)^i\boldsymbol{P}^{i}_{(m\times m)}\right] \tag{8}$$

$\boldsymbol{Q_B}$ is called the integral operator matrix of the block pulse function, and the inverse matrix $\boldsymbol{Q}^{-1}_{\boldsymbol{B}(m\times m)}$ is called the differential operator matrix of the block pulse function.

Because the Haar wavelet matrix $\boldsymbol{H}$ is the orthonormal matrix, the integral matrix of $\boldsymbol{H}$ is given as follows.

$$\boldsymbol{Q_H} = \boldsymbol{H}\boldsymbol{Q}^{\boldsymbol{T}}_{\boldsymbol{B}}\boldsymbol{H}^{-\boldsymbol{1}} = \boldsymbol{H}\boldsymbol{Q}^{\boldsymbol{T}}_{\boldsymbol{B}}\boldsymbol{H}^{\boldsymbol{T}} \tag{9}$$

Similarly, the differential operator matrix of $\boldsymbol{H}$ is given as follows.

$$\boldsymbol{Q}^{-\boldsymbol{1}}_{\boldsymbol{H}} = \boldsymbol{H}(\boldsymbol{Q}^{\boldsymbol{T}}_{\boldsymbol{B}})^{-\boldsymbol{1}}\boldsymbol{H}^{-\boldsymbol{1}} = \boldsymbol{H}(\boldsymbol{Q}^{\boldsymbol{T}}_{\boldsymbol{B}})^{-\boldsymbol{1}}\boldsymbol{H}^{\boldsymbol{T}} \tag{10}$$

IV. Haar Wavelet Expression of Blanch Characteristics of Nonlinear Time Varying Circuit Elements

The general interval $[t_{min}, t_{max})$ is rescaled to interval [0,1) because Haar wavelet function is defined on interval [0,1) . In this paper, if t_{min} is 0, capacitance c[F] and respectively, inductance l[H] can be described as $C = c/t_{max}$, and L=l/t_{max} without losing the generality. Next, we show the Haar wavelet expression of branch characteristics of nonlinear time varying circuit elements for the expression in wavelet domain.

Capacitor :

$$\mathrm{v}(t) = v(0\_) + \frac{1}{C}\int_0^t i(\tau)d\tau, \quad v_0 \coloneqq v(0\_)$$

$$\boldsymbol{V} = \boldsymbol{V_0} + \boldsymbol{C}^{-\boldsymbol{1}}_{\boldsymbol{w}}\boldsymbol{Q_H}\boldsymbol{I}$$

$$\text{or} \quad \boldsymbol{I} = \boldsymbol{C_w}\boldsymbol{Q}^{-\boldsymbol{1}}_{\boldsymbol{H}}[\boldsymbol{V} - \boldsymbol{V_0}] \tag{11}$$

$$\boldsymbol{C_w} = \boldsymbol{Hdiag}[C(i_0,t_0),C(i_1,t_1),\cdots,C(i_{m-1},t_{m-1})]\boldsymbol{H^T}$$

Inductor :

$$i(t) = i(0\_) + \frac{1}{L}\int_0^t v(\tau)d\tau, \quad i_0 \coloneqq i(0\_)$$

$$\boldsymbol{I} = \boldsymbol{I_0} + \boldsymbol{L}^{-\boldsymbol{1}}_{\boldsymbol{w}}\boldsymbol{Q_H}\boldsymbol{V}$$

$$\text{or} \quad \boldsymbol{V} = \boldsymbol{Q}^{-\boldsymbol{1}}_{\boldsymbol{H}}\boldsymbol{L_w}[\boldsymbol{I} - \boldsymbol{I_0}] \tag{12}$$

$$\boldsymbol{L_w} = \boldsymbol{Hdiag}[L(i_0,t_0),L(i_1,t_1),\cdots,L(i_{m-1},t_{m-1})]\boldsymbol{H^T}$$

Resistor :

$$\mathrm{v}(t) = Ri(t)$$

$$\boldsymbol{V} = \boldsymbol{R_w I},\ \boldsymbol{R_w} = \boldsymbol{diag}[\boldsymbol{R}] \tag{13}$$

$$\boldsymbol{R}_w = \boldsymbol{H}\boldsymbol{diag}[R(i_0,t_0), R(i_1,t_1), \cdots, R(i_{m-1},t_{m-1})]\boldsymbol{H^T}$$

V. METHOD TO FIND STEADY-STATE PERIODIC SOLUTIONS

Consider the following ordinary differential equation,

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t) \triangleq \boldsymbol{A}(\boldsymbol{x}, t)\boldsymbol{x} + \boldsymbol{u}(t) \tag{14}$$

where $\boldsymbol{x}(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T \in \boldsymbol{R}^{n\times1}$ is an unknown state variable vector, $\mathrm{A(x,t)} \in \mathrm{R}^{n\times n}$ is a nonlinear time varying parameter matrix, and $\boldsymbol{u}(t) = [u_1(t)\ u_2(t)\ \cdots\ u_n(t)]^T \in \boldsymbol{R}^{n\times1}$ is an external force vector. However, in this paper is $\mathrm{u(t)} = 0$ because the circuit is the autonomous nonlinear circuit. Assume than this system has an unknown period *T*, and that we can find the periodic solution $\boldsymbol{x_p}(t)$ with period *T*, i.e., $\boldsymbol{x_p}(t) = \boldsymbol{x_p}(t+T)$ for all *t*. In order to find the steady-state periodic solution, we should find the solution for the interval [0,1) under the appropriate boundary conditions. For the wavelet expression of the differential equations, we define the discretized expression of $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$ as $\overrightarrow{x_i} = [x_i(t_1)\ x_i(t_2)\ \cdots\ x_i(t_m)]^T \in \boldsymbol{R}^{m\times1}$ and $\overrightarrow{\mathrm{u_i}} = [\mathrm{u_i(t_1)}\ \mathrm{u_i(t_2)}\ \cdots\ \mathrm{u_i(t_m)}]^{\mathrm{T}} \in \mathrm{R}^{m\times1}$ for $i = 1,2,\cdots,m$ respectively.

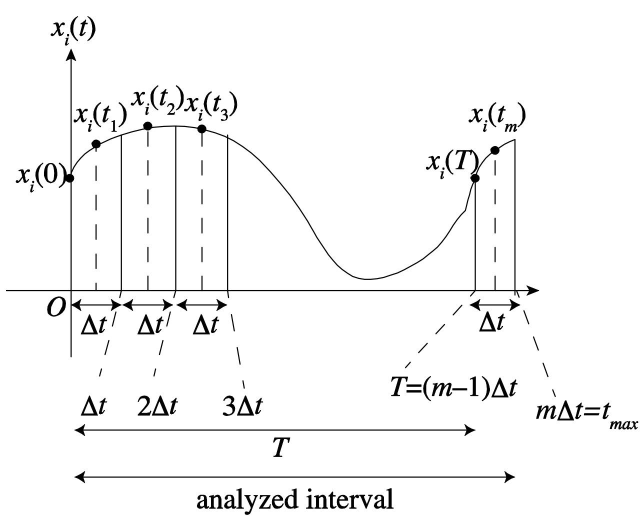

Fig.2: Definition of the analyzed interval and the time step.

The wavelet transformed expression of Eq. (14) can be derived as

$$\boldsymbol{Q_m^{-1}}[\boldsymbol{X} - \boldsymbol{X_0}] = \boldsymbol{A_H X} + \boldsymbol{U} \tag{15}$$

where $\boldsymbol{X} = [(\boldsymbol{H}\overrightarrow{x_1})^{\boldsymbol{T}}\ (\boldsymbol{H}\overrightarrow{x_2})^{\boldsymbol{T}}\ \cdots\ (\boldsymbol{H}\overrightarrow{x_n})^{\boldsymbol{T}}]^{\boldsymbol{T}}$ $\triangleq \left[\boldsymbol{X_1^T}\ \boldsymbol{X_2^T}\ \cdots\ \boldsymbol{X_n^T}\right]^{\boldsymbol{T}} \in \boldsymbol{R}^{\boldsymbol{mn\times1}}$ is an unknown wavelet coefficients vector,

$\boldsymbol{X_0} = [(\boldsymbol{H}\overrightarrow{x_{10}})^T\ (\boldsymbol{H}\overrightarrow{x_{20}})^T\ \cdots\ (\boldsymbol{H}\overrightarrow{x_{n0}})^T]^T \triangleq [\boldsymbol{X}_{10}^T\ \ \boldsymbol{X}_{20}^T\ \cdots\ \boldsymbol{X}_{n0}^T]^T \in \boldsymbol{R}^{mn\times1}$.

$\overrightarrow{x_{\iota0}}$ is an initial value vector. Note that $\overrightarrow{x_{\iota0}}$ is also unknown for this case. Moreover,

$$\boldsymbol{Q_m^{-1}} = \begin{bmatrix} \boldsymbol{Q_H^{-1}} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q_H^{-1}} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{Q_H^{-1}} \end{bmatrix} \in \boldsymbol{R^{mn\times mn}} \tag{16}$$

and, $\boldsymbol{A_H} \in \boldsymbol{R}^{\boldsymbol{mn\times mn}}$ is the wavelet region expression of $\boldsymbol{A}$ derived by the relationships described in Sect. Ⅳ. At this moment, as $\boldsymbol{X}$ and $\boldsymbol{X_0}$ are unknown, we cannot solve this algebraic equations.

To determine the boundary condition, we set the analyzed interval as shown in Fig. 2. The time step $\Delta t = \frac{T}{m-1}$ and $t_{max} = T + \Delta t$. Because of the feature of the matrix $\boldsymbol{Q}_{B(m\times m)}$, time t_i is calculated as $t_i = \frac{\Delta t}{2} + (j-1)\Delta t\ \ (j = 1,2,\cdots,m)$. Due to the periodicity, the relationship $x_i(t_1) = x_i(t_m)$ for all $i = 1,2,\cdots,n$ is derived. From Eq. (5), this relationship is rewritten as follows.

$$[h_{11}\ h_{21}\ \cdots\ h_{m1}]\boldsymbol{X_i} = [h_{1m}\ h_{2m}\ \cdots\ h_{mm}]\boldsymbol{X_i} \tag{17}$$

then,

$$[h_{11} - h_{1m}\ \ h_{21} - h_{2m}\ \cdots\ h_{m1} - h_{mm}]\boldsymbol{X_i} = \boldsymbol{0} \tag{18}$$

Setting $[h_{11} - h_{1m}\ \ h_{21} - h_{2m}\ \cdots\ h_{m1} - h_{mm}] \triangleq \boldsymbol{h_b} \in \boldsymbol{R^{1\times m}}$ and $\boldsymbol{diag}(\boldsymbol{h_b}) \triangleq \boldsymbol{H_b} \in \boldsymbol{R^{n\times mn}}$, the relationship

$$\boldsymbol{H_b X} = \boldsymbol{0} \tag{19}$$

is derived.

To derive the unknown vector $\overrightarrow{x_0}$, we consider the relationship between $\boldsymbol{X}$ and $\boldsymbol{X_0}$. From Eq. (15), we see the matrix $\boldsymbol{Q_H^{-1}X_{i0}}$ from $\boldsymbol{Q_H^{-1}X_0}$. From the relationship $\boldsymbol{X_{i0}} = \boldsymbol{H}\vec{x}_{i0}$,

$$\boldsymbol{Q_H^{-1}X_{i0}} = \boldsymbol{Q_H^{-1}H\vec{x}_{i0}} \tag{20}$$

If we set $\boldsymbol{Q_H^{-1}H} \triangleq [q_{ij}] \in \boldsymbol{R}^{m\times m}$,

$$\boldsymbol{Q_H^{-1}H\vec{x}_{i0}} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mm} \end{bmatrix} \begin{bmatrix} x_i(0) \\ x_i(0) \\ \vdots \\ x_i(0) \end{bmatrix}$$

$$= \begin{bmatrix} q_{11}+q_{12}+\cdots+q_{1m} \\ q_{21}+q_{22}+\cdots+q_{2m} \\ \vdots \\ q_{m1}+q_{m2}+\cdots+q_{mm} \end{bmatrix} \boldsymbol{x_i}(0)$$

$$\triangleq \boldsymbol{q_0 x_i}(0) \qquad (21)$$

Then we define $\boldsymbol{Q_0} = \boldsymbol{diag(q_0)} \in \boldsymbol{R}^{mn\times n}$, Eq.(16) is rewritten as

$$(\boldsymbol{Q_m^{-1}} - \boldsymbol{A_H})\boldsymbol{X} - \boldsymbol{Q_0}\vec{\boldsymbol{x}}_0 = \boldsymbol{0} \qquad (22)$$

From Eqs. (22) and (19), we can derive $\mathrm{n(m-1)}$ -dimensional algebraic equations as follows,

$$\begin{bmatrix} \boldsymbol{Q_m^{-1}} - \boldsymbol{A_H} & -\boldsymbol{Q_0} \\ \boldsymbol{H_b} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{X} \\ \vec{\boldsymbol{x_0}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \qquad (23)$$

In this equation, the number of the unknown of the variables coincides with the dimension of equation because t_{max} is known in nonautonomous nonlinear circuit. Therefore, we can solve the equation by the usual method shown in [5], [7]. However, the number of the unknown variables does not coincide with the dimension of equation because t_{max} is unknown in autonomous nonlinear circuit. Therefore, we cannot solve the equation. Therefore, Eq. (23) is rewritten to the equation of norm as follows to equalize the left-hand side of Eq. (23) to 0.

$$\boldsymbol{F(X)} = \|(\boldsymbol{H_b X})^T \quad \{(\boldsymbol{Q_m^{-1}} - \boldsymbol{A_H})\boldsymbol{X} - \boldsymbol{Q_0}\vec{\boldsymbol{x_0}}\}^T\|_2^2 \qquad (24)$$

We derive the period being *F(X)* =*0* by using the optimization method to minimize the norm of this equation. Then, we can derive the unknown t_{max}, $\boldsymbol{X}$ and $\vec{x_0}$. In this paper, we use Levenberg-Marquardt method as optimization method. Finally, we derive the approximated solution of Eq. (14) from Eq. (5).

VI. EXAMPLE

In this section, we show a simple example to confirm the effectiveness of the proposed method. The van der Pol oscillator shown in Fig. 3 is analyzed in this example. This oscillator has nonlinear voltage controlled current source whose characteristics are as follows.

$$\boldsymbol{f(V_{out})} = \boldsymbol{5}\cdot\left(\boldsymbol{V_{out}} - \boldsymbol{V_{out}^3/3}\right) \qquad (25)$$

The circuit parameter is shown in Table 1 and capacitance c[F] and inductance l[H] are $\boldsymbol{C} = \boldsymbol{c}/t_{max}$ and $\boldsymbol{L}=\boldsymbol{l}/t_{max}$ respectively. The circuit equations are written as follows.

$$\begin{bmatrix} \dot{\boldsymbol{V}}_{\boldsymbol{out}} \\ \dot{\boldsymbol{I}}_{\boldsymbol{L}} \end{bmatrix} = \begin{bmatrix} \frac{\boldsymbol{5\cdot(1-V_{out}^2/3\ )}}{\boldsymbol{C}} & -\frac{\boldsymbol{1}}{\boldsymbol{C}} \\ \frac{\boldsymbol{1}}{\boldsymbol{L}} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{V_{out}} \\ \boldsymbol{I_L} \end{bmatrix} \qquad (26)$$

TABLE I. PARAMETER VALUES FOR VAN DER POL OSCILLATOR.

Parameter	value
Inductance L	1H
Capacitance C	1F

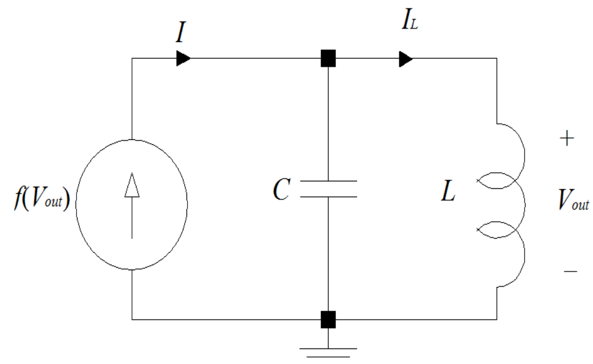

Fig. 3: A van der Pol oscillator.

If we set $\boldsymbol{I} = \boldsymbol{f(V_{out})}$, the Haar wavelet expression of branch characteristics of the current source can be derived from Eq. (25) as

$$\boldsymbol{H}\vec{\boldsymbol{I}} = \boldsymbol{Hdiag}\left[5\left(1-\frac{V_{out1}^2}{3}\right), 5\left(1-\frac{V_{out2}^2}{3}\right), \cdots, 5\left(1-\frac{V_{outm}^2}{3}\right)\right]\overrightarrow{V_{out}} \qquad (27)$$

where $\vec{I}$ and $\overrightarrow{V_{out}}$ are the discretized expression of the current $\boldsymbol{I}$ and the voltage $\boldsymbol{V_{out}}$, respectively. As the matrix $\boldsymbol{H}$ is orthonormal, Eq. (27) can be rewritten as follows.

$$\boldsymbol{H}\vec{\boldsymbol{I}} = \boldsymbol{H}diag\left[5\left(1-\frac{V_{out1}^2}{3}\right), 5\left(1-\frac{V_{out2}^2}{3}\right), \cdots, 5\left(1-\frac{V_{outm}^2}{3}\right)\right]\boldsymbol{H^T H}\overrightarrow{V_{out}} \qquad (28)$$

If we set $\boldsymbol{I}_H = \boldsymbol{H}\vec{\boldsymbol{I}}$ and $\boldsymbol{V}_H = \boldsymbol{H}\overrightarrow{V_{out}}$, and define the matrix G_w as

$$\boldsymbol{G_w} = \boldsymbol{Hdiag}\left[5\left(1-\frac{V_{out1}^2}{3}\right), 5\left(1-\frac{V_{out2}^2}{3}\right), \cdots, 5\left(1-\frac{V_{outm}^2}{3}\right)\right]\boldsymbol{H^T},$$

we can derive the wavelet transformed form of Eq. (25) as

$$\boldsymbol{I}_H = \boldsymbol{G}_w \boldsymbol{V}_H \tag{29}$$

as shown in Eq. (13). Using this relationship, the wavelet expression of Eq. (26) becomes a nonlinear algebraic equation. This equation can be solved by the optimization method such as Levenberg-Marquardt method.

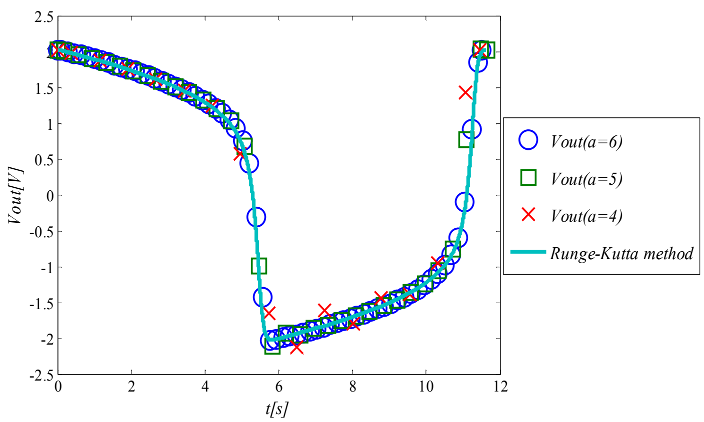

Fig. 4: Calculation results for the proposed method for $a = 4, 5, 6$.

Figure 4 shows the calculation results for the proposed method. The most precise approximation is the case for $a = 6$ in the example. In Fig.4, we can see that the approximation approaches to the calculation result using the Runge-Kutta method as the value of a becomes larger. The proposed method is easier to use than the method shown in [6]. Moreover, by combining with the method shown in [3], the proposed method will be improved by selecting the resolution adaptively around the region where the accuracy is not enough. Therefore, we consider that we can find more accurate result effectively. However, sometimes the proposed method does not converge to the appropriate result according to the initial value of the optimization method. This problem is one of our future works.

VII. CONCLUSION

In this paper, we have proposed the method to derive steady-state periodic solution of the autonomous nonlinear circuit using Haar wavelet transforms, and confirmed its performance using the van der Pol oscillator as an example. Obtaining more precise solution, finding how to set the initial value of the optimization method, the improvement of the algorithm and the substantiate of the proposed method to the more complex circuits seem to be the future works.

REFERENCES

[1] S. Barmada and M. Raugi, "A general tool for circuit analysis based on wavelet transform," Int. J. Circuit Theory, vol.28, no.5, pp.461-480, Apr. 2000.

[2] A. Ohkubo, S. Moro, and T. Matsumoto, "A method for circuit analysis using Haar wavelet transforms," Proc. of IEEE Midwest Symposium on Circuits and Systems (MWSCAS'04), vol.3, pp.399-402, July 2004.

[3] M. Oishi, S. Moro, and T. Matsumoto, "A Method for Circuit Analysis using Haar Wavelet Transform with Adaptive Resolution," Proc. of international Symposium on Nonlinear Theory and its Applications (NOLTA'08), pp.369-372, Sep. 2008.

[4] K. C. Tam, S.-C. Wong, and C. K. Tse, "An improved wavelet approach for finding steady state waveforms of power electronics circuits using discrete convolution," IEEE Trans. Circuits Syst.-II, vol.52, no.10, pp.690-694, Oct. 2005.

[5] S. Moro, "Analysis Method for Steady-State Periodic Solutions in Periodically Driven Nonlinear Circuits using Haar Wavelet Transform," Proc. of International Symposium on Nonlinear Theory and its Applications (NOLTA2012), pp.316-319, Oct. 2012.

[6] X. Lin, B.Hu, X. Ling, and X. Zeng, "A Wavelet-Balance Approach for Steady-State Analysis of Nonlinear Circuits," IEEE Trans. Circuits Syst.-I, vol.49, no.5, pp689-694, May 2002.

[7] M. Mochizuki and S. Moro, "Steady-state analysis using Haar wavelet transform in power electronics circuits including nonlinear elements," Proc. of International Symposium on Nonlinear Theory and its Applications, pp.483-486, Sept. 2013.

[8] J. L. Wu, C. H. Chen, and C. F. Chen, "Numerical inversion of Laplace transform using Haar wavelet transforms," IEEE Trans. Circuits Syst.-I, vol.48, no.1, pp.120-122, Jan. 2001.

[9] C. F. Chen, Y. T. Tsay, and T. T. Wu, "Walsh operational matrices for fractional calculus and their application to distributed systems," J. Franklin Institute, vol.303, no.3, pp.267-284, Mar. 1977.