Torus and Spatio-Temporal Patterns in Coupled Oscillators Networks with Hard-Type Nonlinear Characteritics

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Torus and Spatio-Temporal Patterns in Coupled Oscillators Networks with Hard-Type Nonlinear Characteritics

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Abstract—In this study, we investigate the oscillation phenomena in a random-coupled network of hard-type oscillators whose nonlinear characteristics are represented by fifth-power polynomials. Recently, we have shown that the chaotic phenomena are seen in the network with soft-type oscillators whose nonlinear characteristics are represented by third-power polynomials. In this paper, for the networks with hard-type oscillators, we show that not only chaotic phenomena but also torus and spatio-temporal patterns such as amplitude switching are observed by changing the coupling coefficients.

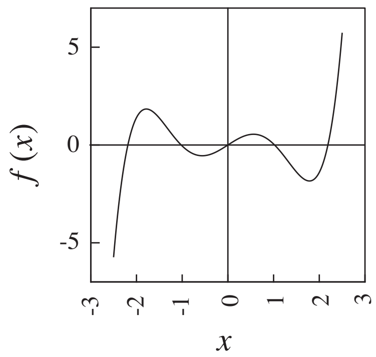

Figure 1: Nonlinear characteristics represented by fifth-power polynomial.

1. Introduction

In recent years, various oscillation and synchronization phenomena have been investigated for coupled oscillators [1]–[9]. We have proposed star-coupled oscillators in which N oscillators are coupled by one resister [2]. Because the current through the coupling resister should be reduced to a minimum, N-phase synchronization phenomena can be observed. It is considered that this coupled oscillator can be used in various fields, because a variety of synchronization phenomena are exhibited. Especially, it is considered that star-coupled oscillators which are arranged in lattice or hexagonal structure can be used as some kinds of cellular neural networks [3].

On the other hand, there are not only regular networks like lattice or hexagonal shape, but also complex networks like small-world or scale-free network [10]–[12]. The scale-free networks have features that most nodes has very few connections but a small number of particular nodes has many connections. From this feature, even if some parts of most nodes which have very few connections are removed, a global connection in a network is preserved. However, if small number of particular nodes which have many connections are removed, a network is interrupted simply. That is, they are robust to random removal, but they are vulnerable when the most connected nodes are removed. These features of scale-free networks are shown in many networks of various fields, e.g., internet, process diffused of word of mouth, metabolic network, etc. That is, to analyze the features of scale-free networks may be used for analyzing those practical networks.

From these points of view, we have studied about oscillation phenomena of a scale-free coupled oscillators network. We have composed a scale-free network called Barabási-Albert model by resistively coupled van der Pol oscillators, and confirmed the oscillation phenomena in the proposed network by both numerical calculation and circuit experiments. As a result, we have shown that intermittency chaos is observed in the proposed network [9].

In this study, we investigate the oscillation phenomena in a random-coupled network based on Barabási-Albert model with hard-type oscillators whose nonlinear characteristics are represented by fifth-power polynomials. For hard-type oscillators, we can control if they oscillate or not by changing the initial states, while soft-type oscillators oscillate from any initial conditions. Therefore, it is reported that a great variety of phenomena has been seen in the coupled networks of hard-type oscillators [6]–[8]. In this paper, we show that not only chaotic phenomena but also torus and spatio-temporal patterns such as amplitude switching are observed by changing the coupling coefficients.

2. Hard-Type Oscillators and Their Coupled Networks

A hard-type oscillator is consist of an LC tank circuit and a nonlinear negative conductance whose characteristics are represented by the fifth-power polynomials as shown in Fig. 1. Because an ordinary van der Pol oscillator has

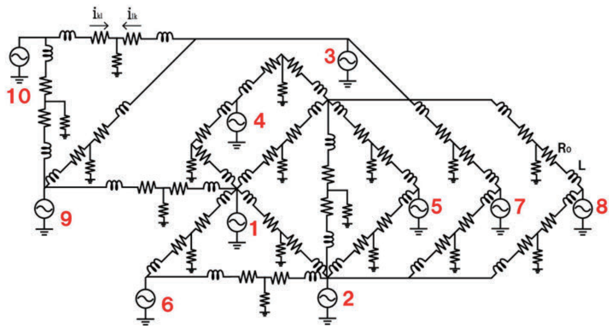

Figure 2: Coupled oscillators network.

third-power nonlinear characteristics, so-called "soft-type" oscillator, the origin becomes unstable equilibrium point, and a stable limit cycle is observed. Therefore, it oscillates from any initial conditions. On the other hand, for a "hard-type" oscillator, the origin becomes a stable equilibrium point due to the positive slope around the origin as shown in Fig. 1. Therefore, it has both a stable equilibrium point and a stable limit cycle and we can chose if it will oscillate or not by changing the initial states. In the coupled networks with hard-type oscillators, various oscillation phenomena which are not shown in the networks with soft-type oscillators have been reported [2], [6]–[8].

3. Barabási-Albert Model

Barabási-Albert model is scale-free network model proposed by Barabási and Albert [11, 12]. This model is constructed by growth and preferential attachment. A new node having n links is added to initial network step by step with the probability Π_i. Π_i is the probability when the node i is selected to the destination of a new link, and decided by the following equation,

$$\Pi_i = \frac{N_i}{\sum_j N_j} \qquad (1)$$

where i and j are node numbers in the network and N_i is a number of links connected to node i.

4. Circuit Models

For $m_0 = 2$ and $n = 2$, we construct a scale-free network by Barabási-Albert model based on the coupled oscillators shown in Ref. [1]. Figure 2 shows the circuit model for this paper. Table 1 shows the number of links connected to each node decided by Eq. (1).

The normalized circuit equation of this network is described as

$$\begin{aligned} \dot{x}_k &= -\sum_l y_{kl} - \varepsilon\left(x_k - \frac{1}{3}\beta {x_k}^3 + \frac{1}{5}{x_k}^5\right) \\ \dot{y}_{kl} &= x_k - \gamma y_{kl} - \alpha(y_{kl} + y_{lk}) \qquad (2) \\ (k &= 1, 2, \cdots, 10, \quad l = 1, 2, \cdots, 10) \end{aligned}$$

Table 1: Number of links of each oscillator in the network.

node i	1	2	3	4	5	6	7	8	9	10
# of links	5	6	8	2	2	2	2	2	3	2

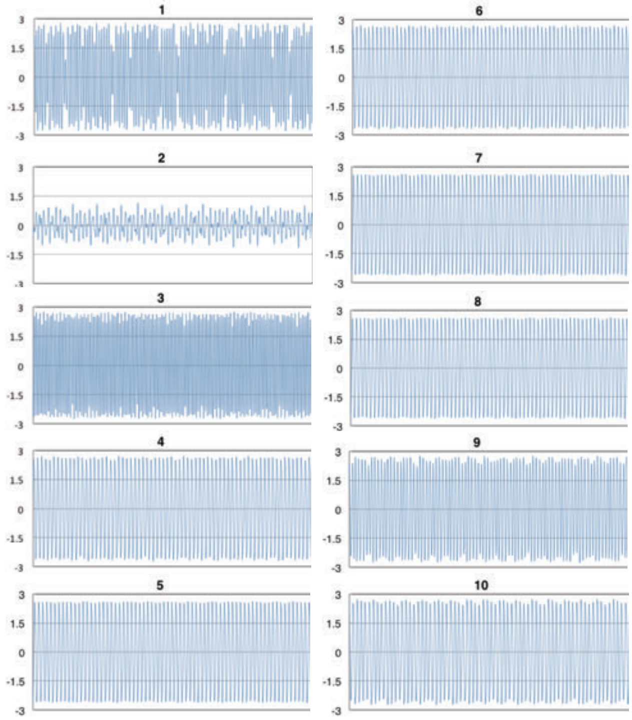

Figure 3: Time waveform for $\alpha = 0.5$.

where $\dot{x} = dx/d\tau$. τ is normalized time, α is the coupling factor, ε is the strength of nonlinearity, β is the parameter which decides the amplitude, and γ is the internal resistance of the inductors. In the following section, we show the numerical calculation of the state equation Eq. (2).

5. Numerical Results

In this section, numerical calculations of Eq. (2) using fourth order Runge-Kutta method are carried out. In the following results, we fix the parameters $\beta = 4.5$, $\varepsilon = 0.5$, $\gamma = 0.026$, and take different α as $\alpha = 0.5, 1.0, 3.0, 5.0$.

First, we show the time waveforms for each α in Figs. 3–6. In each result, we can observe the aperiodic oscillations. From Figs. 3–5, it is shown that both large amplitude oscillations and small amplitude oscillations are seen while all of the oscillation amplitudes are almost equal in the network with soft-type oscillators [9]. In particular, for $\alpha = 3.0$, the amplitudes randomly switch between large and small ones. It is considered that it exhibits spatio-temporal patterns for some parameters, and it has not seen yet in the networks with soft-type oscillators.

Figures 7–10 show the Poicaré maps for the section $x_1 =$

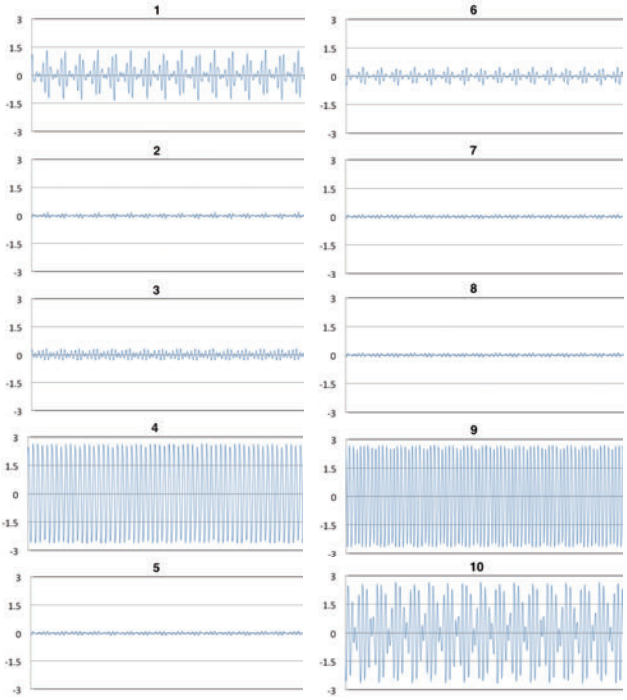

Figure 4: Time waveform for $\alpha = 1.0$.

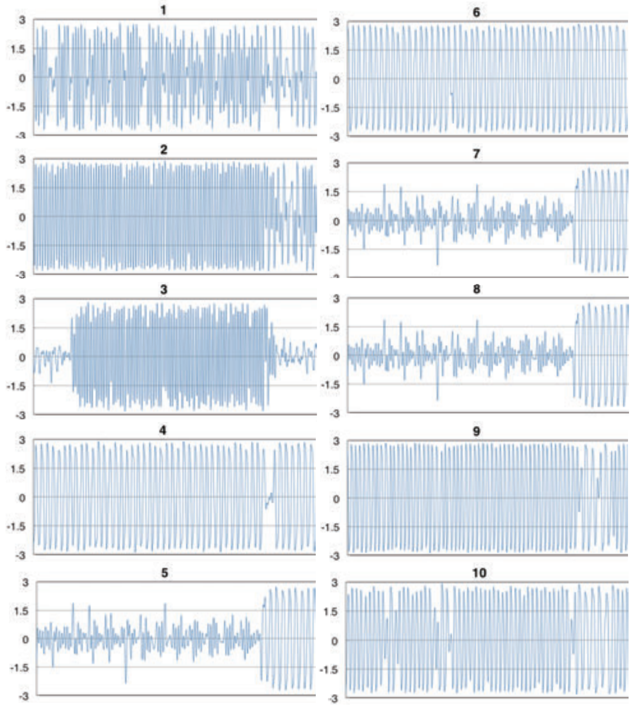

Figure 5: Time waveform for $\alpha = 3.0$.

0, $\dot{x}_1 > 0$. From these figures, it is proved that the chaotic oscillations are observed except the case of $\alpha = 1.0$. In Fig. 8, for the case of $\alpha = 1.0$, it is shown that the torus has been observed since the maps draw the closed curves. Also, torus has not seen in the networks with soft-type oscillators.

From the numerical results, we can see the torus and the statio-temporal chaos which have never be observed in the networks with soft-type oscillators. It means the proposed networks with hard-type oscillators have great potential to model and analyze the practical dynamical networks.

6. Conclusions

In this paper, we have studied oscillation phenomena in coupled oscillators network with hard-type oscillators using Barabási-Albert model. In this system, we can see not only intermittency chaos but also torus and spatio-temporal chaos which have never seen in the networks with soft-type oscillators by changing the coupling coefficients. We believe that the proposed networks with hard-type oscillators have great potential to model and analyze the practical dynamical networks. More precise analysis of the bifurcation and application of the networks are our future problems.

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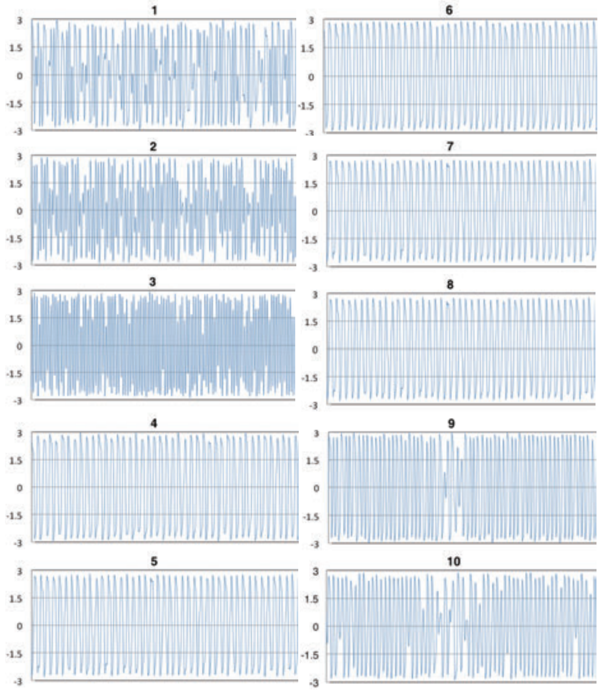
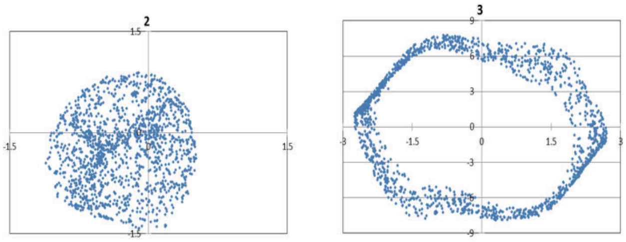


Figure 6: Time waveform for $\alpha = 5.0$.



(a) (b)

Figure 7: Poincaré map for $\alpha = 0.5$. (a) x_2, (b) x_3.

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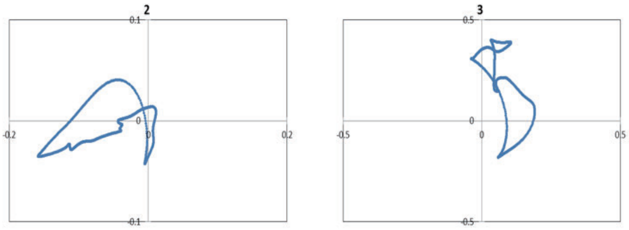
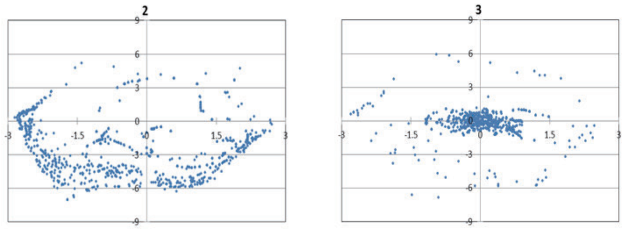


(a) (b)

Figure 8: Poincaré map for $\alpha = 1.0$. (a) x_2, (b) x_3.



(a) (b)

Figure 9: Poincaré map for $\alpha = 3.0$. (a) x_2, (b) x_3.

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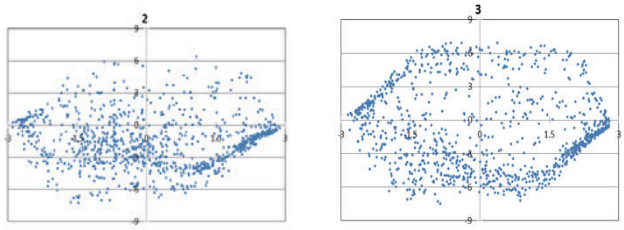

(a) (b)

Figure 10: Poincaré map for $\alpha = 5.0$. (a) x_2, (b) x_3.