Distribution of the Number of Arms of (PSt)f-n-star-(PIs)n Hetero-Star Block Copolymers Prepared via Anionic Living Polymerization of Macromonomers
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Distribution of the Number of Arms of (PSt)_{m-star}-(PIs)_n Hetero-Star Block Copolymers Prepared via Anionic Living Polymerization of Macromonomers

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Abstract

The distributions of the number of arms (DNA) of (polystyrene)$_{6.2}$-star-polyisoprene ([PSt]$_{6.2}$-star-PIs) and (PSt)$_{7.6}$-star-(PIs)$_{6.3}$ hetero-star block copolymers and its precursors prepared via anionic living polymerization of macromonomers have been determined by plotting a weight fraction of the number of arms against the number of arms. Three-dimensional molecular weight distributions (3D-MWD) of the two hetero-star block copolymers and its precursors were also determined by plotting a weight fraction at the $i$th elution volume against the corresponding molecular weight ($M_i$) and $i$th elution volume using GPC in conjunction with a low-angle laser light-scattering detector, a refractive index detector, and an ultraviolet detector. A non-linear relationship between the $i$th elution volume and $M_i$ of (PSt)$_{6.2}$-star-PIs was observed and explained from a $g_{star}$ perspective. The present paper describes the analytical method for determination of the DNA using the MWD. The DNAs of the six samples were found to be narrow and similar to the MWDs of the corresponding samples. The synthetic mechanism of the hetero-star block copolymers was discussed from a DNA perspective.
1. Introduction

We have previously reported the preparation, some physical properties, and morphological behaviors of \((A)_{f,1}\)-star-B type hetero-star block copolymers via anionic living polymerization of (4-vinylbenzyl)polystyrene macromonomer (PStM) and (4-vinylbenzyl)polyisoprene macromonomer (PIsM) [1, 2]. When we submitted the manuscript, one of the reviewers pointed out the importance of examining the distribution of the number of arms (DNA) of hetero-star block copolymers, even though the molecular weights of arms \(M_n^{\text{arm}}\) and final hetero-star block copolymers \(M_n^{\text{star}}\) were reported and the average number of arms \(<f>\) had been calculated as \(<f> = M_n^{\text{star}} / M_n^{\text{arm}}\). The reason for examining the distribution might be that even if the \(M_n^{\text{arm}}, M_n^{\text{star}}\) and \(<f>\) values of two hetero-star block copolymers are the same as each other, some of their physical properties may differ when their DNAs differ [3, 4]. However, the DNA of \((A)_{f,1}\)-star-B has been difficult to resolve, and this problem has until now remained unresolved.

To our knowledge, there has been no study of the DNA of star polymers and hetero-star block copolymers [5-12]. However, we recently found a special means of analysis to estimate the DNA of the \((PSt)_{f,1}\)-star-PIs hetero-star block copolymer [13]. In recent years a method has been developed for the preparation of \((PSt)_{f,a}\)-star-(PIs)\(_a\) hetero-star block copolymers [14] via anionic living polymerization of PStM and PIsM macromonomers. The present paper describes the analytical method, its results and the synthetic mechanism of the \((PSt)_{f,1}\)-star-PIs and \((PSt)_{f,a}\)-star-(PIs)\(_a\) hetero-star block copolymers from a DNA perspective.

2. Experimental

2.1 Polymers

PStM and PIsM macromonomers were prepared as described previously [1, 2]. The \((PSt)_{6.2}\)-star-PIs was prepared by sequential anionic living copolymerization of PStM and isoprene by sec-BuLi in benzene (Bz) at room temperature. The \((PSt)_{7.6}\)-star-(PIs)\(_{6.3}\) was prepared by sequential anionic living copolymerization of PStM and PIsM by \(n\)-BuLi in tetrahydrofuran (THF) at -78°C. Polymerization was carried out in a sealed glass apparatus.
under a pressure of $10^6$ mmHg. The polymerization techniques were almost the same as those employed in previous studies [1, 2].

2.2 Molecular Characterization

The molecular characteristics of two macromonomers and two hetero-star block copolymers of (PSt)$_{f_1}$-star-PIs and (PSt)$_{f_n}$-star-(PIs)$_n$ are described in Table 1; these characteristics are probably the same as those reported elsewhere [1]. All polymer samples were tested on gel permeation chromatography (GPC; Model CCPD, Tosoh Co., Tokyo Japan) in conjunction with a low-angle laser light-scattering (LALLS) detector (Model LS-8000, Tosoh Co.), a refractive index (RI) detector (RI-8010, Tosoh Co.) and an ultraviolet (UV) detector (UV-8011, Tosoh Co.). For the measurements, four high-resolution columns (G2500H, G3000H, G4000, and GMH-M, 7.8 mm i. d. x 60 cm; Tosoh Co.) were connected in a series. A special analysis was required to determine the number-average molecular weight ($M_n$) and the weight-average molecular weight ($M_w$) of the two hetero-star block copolymers using GPC-LALLS. The details have been reported elsewhere [15]. All calculations for determining and drawing the distribution of the number of arms (DNA) were carried out on a personal computer using commercially available software.

3. Results and Discussion

3.1 Molecular weight distributions (MWD)

The RI intensity ($H_{i}^{RI}$) and the LALLS intensity ($H_{i}^{LS}$) of the corresponding RI and LALLS chromatograms of (PSt)$_{f_1}$, (PSt)$_{f_1}$-star-PIs, and (PSt)$_{f_n}$-star-(PIs)$_n$ at the $i$th elution volume are described as follows:

$$H_{i}^{RI} = k_{RI} \left( \frac{dn}{dc} \right) C_i$$

$$H_{i}^{LS} = k_{LS} \left[ \left( \frac{dn}{dc} \right) C_i \right]^2 M_i C_i$$

where $(dn/dc)_i$, $C_i$, and $M_i$ are the refractive index increment, concentration, and molecular weight at the $i$th elution volume.
weight at the \(i\)th elution volume, respectively. From the ratio of the square of Eq. (1) to Eq. (2), the \((dn/dc)\), was eliminated to yield \(M_i\). The resultant \(M_i\) is as follows:

\[
M_i = \left[ \left( \frac{H_i^{LS}}{H_i^{RI}} \right)^2 \right] \left[ k_{LS} / (k_{RI})^2 \right]^{-1} C_i
\]

The instrumental factor of \([k_{LS} / (k_{RI})^2]^{-1}\) can be determined for a series of standard PSTs with several concentrations.

It has been known that the molecular weight of the block copolymer having composition distribution could not be determined by light scattering. The reason is that the \((dn/dc)\), of the block copolymers having the composition distribution could not be determined precisely. However, this problem has been resolved using GPC-LALLS [1, 15]. Thus the \(M_i\) was calculated by considering the composition distribution to become the same equation as Eq. (3). The abstract for deriving Eq. (3) for the block copolymers is described in Appendix A.

The RI intensity of \((\text{PSt})_{i} \text{-} \text{star} \text{-} (\text{PIs})_{n}\) at the \(i\)th elution volume \((H_i^{RI})\) is the summation of the RI intensities of the \((\text{PSt})_{i,\text{a}}\) block \((H_i^{RI}_{\text{PSt}})\) and the \((\text{PIs})_{n}\) block \((H_i^{RI}_{\text{PIs}})\). The UV intensities at the \(i\)th elution volume \((H_i^{UV})\) are described by the same formula as that of the RI intensities.

\[
H_i^{RI} = H_i^{RI}_{\text{PSt}} + H_i^{RI}_{\text{PIs}} = k'_{\text{PSt}} C_i^{\text{PSt}} + k'_{\text{PIs}} C_i^{\text{PIs}}
\]

\[
H_i^{UV} = H_i^{UV}_{\text{PSt}} + H_i^{UV}_{\text{PIs}} = \varepsilon_{\text{PSt}} C_i^{\text{PSt}} + \varepsilon_{\text{PIs}} C_i^{\text{PIs}}
\]

From these equations, the concentrations of the \((\text{PSt})_{i,\text{a}}\) block \((C_i^{\text{PSt}})\) and the \((\text{PIs})_{n}\) block \((C_i^{\text{PIs}})\) at the \(i\)th elution volume can be determined using the \(k'_{\text{PSt}}, k'_{\text{PIs}}, \varepsilon_{\text{PSt}}, \text{and} \varepsilon_{\text{PIs}}\) values: these four instrumental constants could previously be determined by GPC measurements of the PST and PIs homopolymers using the RI and UV detectors. Therefore, the concentration of the \((\text{PSt})_{i} \text{-} \text{star} \text{-} (\text{PIs})_{n} (C_i)\) at the \(i\)th elution volume can be calculated as follows [16]:

\[
C_i = C_i^{\text{PSt}} + C_i^{\text{PIs}}
\]

\[
= \frac{\left( \varepsilon_{\text{PSt}} - \varepsilon_{\text{PIs}} \right) H_i^{RI} + \left( k'_{\text{PIs}} - k'_{\text{PSt}} \right) H_i^{UV}}{k'_{\text{PIs}} \varepsilon_{\text{PSt}} - k'_{\text{PSt}} \varepsilon_{\text{PIs}}}
\]
A combination of Eq. (3) and Eq. (6) provides the $M_i$ value by using three detectors of GPC-LALLS measurements.

On the other hand, a weight fraction of $(\text{PSt})_{f-n}^{star}$-$(\text{PIs})_{n}^{hetero}$-star block copolymers ($W_i^{obs}$) at the $i$th elution volume can be calculated using Eq. (6) as

$$W_i^{obs} = C_i / \Sigma_i C_i$$  

(7)

3.2. Three-dimensional molecular weight distributions (3D-MWD)

The three-dimensional MWDs (3D-MWD) of $(\text{PSt})_{f-1}^{star}$-$(\text{PIs})_{n}$ (Fig. 1) and $(\text{PSt})_{f-2}^{star}$-$(\text{PIs})_{n}$ (Fig. 2) were determined by plotting $W_i^{obs}$ against $M_i$ and the $i$th elution volume. Plots of $W_i^{obs}$ against $M_i$ are well known as a MWD and plots of $M_i$ against the $i$th elution volume are well known as a calibration curve of GPC measurements. As shown in Fig. 1, during polymerization the 3D-MWD of $(\text{PSt})_{4.7}$ shifted to that of $(\text{PSt})_{6.2}$ accompanying an increase in $M_n$ and the constancy of the elution volume. Then, the 3D-MWD of $(\text{PSt})_{6.2}$ shifted to that of $(\text{PSt})_{n}^{star}$-$(\text{PIs})_{6.2}^{hetero}$-star block copolymers accompanying an increase in the $M_n$ and a decrease in the elution volume.

Particular attention should be directed to a non-linear relationship between the $i$th elution volume and $M_i$ observed on the lower $M_n$ sides of $(\text{PSt})_{n}^{star}$-$(\text{PIs})$: namely, there exist many combinations where the different elution volumes have the same $M_n$ values. This non-linear relationship between the elution volume and $M_i$ has never been observed in common linear polymers. As the number of arms for the star polymers increases, the elution volume does not change significantly. This behavior is known as the $g_{star}$ value, defined as $g_{star} = \langle S^2 \rangle_{star}/\langle S^2 \rangle_{linear} = (3f-2)/f^2$, where $\langle S^2 \rangle$ is the mean-square radius of gyration [17, 18]. Let us suppose two $(\text{PSt})_{f-1}^{star}$-$(\text{PIs})_n$ hetero-star block copolymers to have the same $M_n$. One has a higher $M_n^{star}$ of $(\text{PSt})_{f-1}$ and a lower $M_n^{PIs}$ of PIs, while another has a lower $M_n^{star}$ of $(\text{PSt})_{f-1}$ and a higher $M_n^{PIs}$ of PIs. The molecular dimensions of the two $(\text{PSt})_{f-1}$ might be the same, while those of the two PIs differ. Therefore, the two $(\text{PSt})_{f-1}^{star}$-$(\text{PIs})_n$ with the same $M_n$ values appear at different elution volumes. Thus, the non-linear relationship between the elution volume and $M_i$ results in the $M_n^{PIs}$ value of the PIs arm being 6 times higher than that of a PSt arm. In contrast, in the case of $(\text{PSt})_{n}^{star}$-$(\text{PIs})_{n}^{hetero}$ where the $M_n$ values of the PSt
arms are probably the same as those of the PIs arms, a relatively linear relationship was observed, as shown in Fig. 2.

3.3 Distributions of the number of arms (DNA)

A MWD of the polystyrene macromonomer (PStM) can be determined by GPC measurements using standard polystyrenes. The MWD is expressed as \([M_{1,i}~W_{1,i}]\), where \(M_{1,i}\) is the molecular weight at the \(i\)th elution volume in the GPC measurements and is plotted as the abscissa at 200 g mol\(^{-1}\) intervals, and \(W_{1,i}\) is the weight fraction at \(M_{1,i}\) determined by \(W_{f,i} = H_{f}^{RI} / \Sigma H_{f}^{RI}\) and is plotted as the ordinate. The MWD can be described in a (28, 3)-type matrix, where the 28 lines range from \(M_{1,1} = 2800\) to \(M_{1,28} = 8200\), and 3 rows consist of \(i, M_{1,i}\), and \(W_{1,i}\). A MWD of a dimmer of PStM, \([M_{2,i}~W_{2,i}]\) can be calculated using \(M_{2,i} = \Sigma w_{i,j,k} (M_{1,j} + M_{1,k})\) and \(W_{2,i} = \Sigma w_{i,j,k} (W_{1,j} W_{1,k})\), where \(\Sigma w_{i,j,k}\) is the summation of the corresponding term for a combination of \((j, k)\) under a condition of \(i = j + k\). That is, the probability that the first PStM with \(M_j\) and the second PStM with \(M_k\) combine directly to produce the dimmer with \(M_{i,j,k}\) is assumed to be independent of \(M_j\) and \(M_k\) of both these PStMs. The resultant MWD can be described in a (55, 3)-type, matrix, where the 55 lines range from \(M_{2,1} = 5600\) to \(M_{2,55} = 16400\), and 3 rows consist of \(i, M_{2,i}\), and \(W_{2,i}\). Thus, a MWD of a \((\text{PSt})_i\) star polymer, \([M_{f,i}~W_{f,i}]\) can be calculated using \(M_{f,i} = \Sigma w_{f,j,k} (M_{f,j} + M_{f,k})\) and \(W_{f,i} = \Sigma w_{f,j,k} (W_{f,j} W_{f,k})\), where \(f\) is the number of arms corresponding to a degree of polymerization of PStM. The resultant MWD of \((\text{PSt})_i\) can be described in a (28\(f\)-1, 3)-type matrix, and each MWD of \((\text{PSt})_i\) \((f = 1 - 20)\) is shown in Fig. 3. As shown in Fig. 3, each MWD of \((\text{PSt})_i\) is narrow and the distribution broadens with increasing \(f\) values. This behavior was expected before the calculation.

A MWD of a \((\text{PSt})_{\text{cal}}\) star polymer, \([M_{i}~W_{i}]^\text{cal}\) can be calculated using \(M_i = \Sigma w_i M_{f,i}\) and \(W_i = \Sigma w_i W_{f,i}\), where \(<f>\) is the average number of arms and \(w_i\) is the weight fraction of a star polymer having \(f\) arms. On the other hand, a MWD of a \((\text{PSt})_{\text{obs}}\) star polymer, \([M_{i}~W_{i}]^\text{obs}\) can be determined by the GPC-LALLS measurements. By comparing the resultant \([M_{i}~W_{i}]^\text{cal}\) with \([M_{i}~W_{i}]^\text{obs}\), DNA, \([f~w_j]\) for the \((\text{PSt})_{\text{cal}}\) star polymers was determined.

In the case of a \((\text{PSt})_{i,1}\)-star-PIs hetero-star block copolymer, an additional calculation is necessary to estimate a DNA. The PIs weight fraction of \((\text{PSt})_{i,1}\)-star-PIs \((w_{i}^{\text{Pis}})\) at the \(i\)th elution volume can be determined as
\[ w_i^{\text{Pls}} = C_i^{\text{Pls}} / (C_i^{\text{Pls}} + C_i^{\text{Pis}}) \]
\[ = (\epsilon_{\text{Pis}} H_i^{\text{RI}} - k_{\text{Pis}}' H_i^{\text{UV}}) / [(\epsilon_{\text{Pis}} - \epsilon_{\text{Pls}}) H_i^{\text{RI}} + (k_{\text{Pls}}' - k_{\text{Pis}}') H_i^{\text{UV}}] \quad (8) \]

From Eq. (3) and Eq. (8), the molecular weight of the PIs arm \((M_i^{\text{Pls}})\) of the \((\text{PSt})_{f,1}-\text{star}-\text{Pis}\) at the \(i\)th elution volume can be calculated as \(M_i^{\text{Pls}} = M_i w_i^{\text{Pls}}\). A PIs weight fraction of the PIs arm \((W_i^{\text{Pls}})\) at the \(i\)th elution volume can also be calculated as \(W_i^{\text{Pls}} = w_i^{\text{Pls}} / \Sigma w_i^{\text{Pls}}\). Hence, the MWD of a PIs arm, \([M_i^{\text{Pls}} \sim W_i^{\text{Pls}}]\) was estimated. As a MWD of a \((\text{PSt})_{f,1}\) star polymer, \([M_{f,1} \sim W_{f,1,i}]\) has already been calculated, a MWD of \((\text{PSt})_{f,1}-\text{star}-\text{Pis}\), \([M_{f,i} \sim W_{f,i}]\) can be calculated as follows: \(M_{f,i} = \Sigma w_i^{\text{Pls}}(M_{f,1,j} + M_i^{\text{Pls}})\) and \(W_{f,i} = \Sigma w_i^{\text{Pls}} W_{f,1,i}^{\text{Pls}}\). A MWD of a \((\text{PSt})_{f,15}-\text{star}-\text{Pis}\) hetero-star block copolymer, \([M_{f-i} \sim W_{f,i}]^{\text{cal}}\) can be calculated using \(M_i = \Sigma w_i M_{f,i}\) and \(W_i = \Sigma w_i W_{f,i}\). On the other hand, a MWD of the \((\text{PSt})_{f,15}-\text{star}-\text{Pis}\), \([M_{f-i} \sim W_{f,i}]^{\text{obs}}\) can be determined by GPC-LALLS measurements. By comparing the resultant \([M_{f-i} \sim W_{f,i}]^{\text{cal}}\) with \([M_{f-i} \sim W_{f,i}]^{\text{obs}}\), a DNA of the \((\text{PSt})_{f,15}-\text{star}-\text{Pis}\) hetero-star block copolymer was determined. The result is shown in Fig. 4, where a GPC chromatogram and a MWD are also shown.

As shown in Fig. 4, a difference in the elution volume between \((\text{PSt})_{4,1}^{+}\) and \((\text{PSt})_{6,2}^{+}\) was detected with difficulty, but a difference in the MWD between them was detected with ease. The MWD is better than the GPC chromatogram for discussing the molecular characteristics of the \((\text{PSt})_{f,1}\) star polymers. The DNAs of the three samples seem to be similar to the MWDs of the corresponding samples, even though the abscissas of both \(M_i\) and \(f\) are different from each other. It should be noted that it is a mistake to convert the abscissa of the MWD into that of the DNA using an equation of \(f = M_i/M_n^{\text{arm}}\). The \(M_n^{\text{arm}}\) value has a distribution of \(M_i\) and is not constant, although the average \(M_n^{\text{arm}}\) value is a constant; in other words, the equal of \(f = M_i/M_n^{\text{arm}}\) is not correct. If \(f = M_i/M_n^{\text{arm}}\) is correct, the \(f\) value should not be a natural number and the special calculations performed in the present study would not be necessary to estimate the DNA. In conclusion, it is a new finding and has never been reported that the DNAs of the three samples are similar to the MWDs of the corresponding samples.

Each MWD of the three samples appears to be broad, but the distributions are actually narrow because the corresponding \(M_n/M_n^{\text{arm}}\) values are less than 1.09. On the other hand, there is no standard for estimating the sharpness of the DNA. For example, a number-average number of arms \(f_n\) and a weight-average number of arms \(f_w\) can be calculated as \(f_n = \Sigma n_i f_i\)
and $f_w = \sum_j w_j f_j$ using the DNA obtained, and these values are similar to $M_n$ and $M_w$.

However, the $f_w$ has no physical meaning and hence the $f_w / f_n$ values, which are calculated to be less than 1.09 also have no physical meaning. So, a measure of the sharpness of the DNA is necessary. A normalized difference between the average number of arms and the actual number of arms ($R_f$) might be proposed as a measure of the DNA.

$$R_f = 100 \sum_j w_j \left| f_j - <f> \right| / <f>$$  (9)

The (100-$R_f$)% value is the probability that the (PSt)$_{cf\cdot}$ and (PSt)$_{cf\cdot1\cdot5\cdot}$star-PIs star polymers can be, respectively, considered as the homogeneous (PSt)$_f$ and (PSt)$_{f\cdot}$star-PIs star polymers without MWDs. The (100-$R_f$)% values of the three samples were found to be more than 80%. The DNAs in Fig. 4 might be concluded to be narrow.

The DNA of the (PSt)$_{4\cdot7}$ shifted to that of the (PSt)$_{6\cdot2}$ in parallel and the DNA of the (PSt)$_{6\cdot2}$ also then shifted to that of the (PSt)$_{6\cdot2\cdot}$star-PIs in parallel. These findings suggest that the anionic living polymerization of PStM and Is monomer by (PSt)$_f$ carbanions proceeded homogeneously. That is, a selective polymerization where PStM and Is are polymerized predominantly by the (PSt)$_f$ having the lower $M_n$ did not occur under the present polymerization conditions.

In the case of the (PSt)$_{i\cdot}$star-(PIs)$_n$ hetero-star block copolymer, the calculation for determining $[M_{f\cdot}W_{f\cdot}]$ became too complicated because it is common for both $M_n$ and $M_w/M_n$ values of PStM to be different from those of PI'sM. However, the $M_n$ values of PStM and PI'sM are 5.45x10$^3$ and 5.33x10$^3$, respectively. The $M_n/M_w$ values of PStM and PI'sM are 1.03 and 1.04, respectively. Thus, all arms of the (PSt)$_{i\cdot}$star-(PIs)$_n$ can be considered to be the same as each other when calculating MWD. These conditions had been set to perform the present calculation with ease. Therefore, the MWD of a (PSt)$_{i\cdot}$star-(PIs)$_n$ hetero-star block copolymer, $[M_{f\cdot}W_{f\cdot}]$ can be calculated using $M_{f\cdot} = \sum_{i=j+k} (M_{f\cdot1\cdot} + M_{1\cdot})$ and $W_{f\cdot} = \sum_{i=j+k} (W_{f\cdot1\cdot} W_{1\cdot})$.

The MWD of a (PSt)$_{cf\cdot}$star-(PIs)$_\infty$, $[M_{f\cdot}W_{f\cdot}]$ can be calculated using $M_{f\cdot} = \sum_j w_j M_{f\cdot}$ and $W_{f\cdot} = \sum_j w_j W_{f\cdot}$. By comparing the resultant $[M_{f\cdot}W_{f\cdot}]$ with $[M_{f\cdot}W_{f\cdot}]$ determined by GPC-LALLS measurements, DNA of the (PSt)$_{cf\cdot}$star-(PIs)$_\infty$ hetero-star block copolymer was determined. The result is shown in Fig. 5, where a GPC chromatogram and a MWD are also shown.
As shown in Fig. 5, a difference in the elution volume between (PSt)_{7.6}Star-(PIs)_{4.6} and (PSt)_{7.6}Star-(PIs)_{6.3} was detected with difficulty, but a difference in the MWD between them was detected with ease. The DNAs of the three samples seem to be similar to the corresponding MWDs. This is a new finding and has never been reported.

Each MWD of the three samples appears to be broad, and the MWD of (PSt)_{7.6}Star-(PIs)_{6.3} seems especially broad compared to that of (PSt)_{7.6}. However, the $M_i$ range ratio from $5 \times 10^4$ to $11 \times 10^4$ of (PSt)_{7.6}Star-(PIs)_{6.3} is probably the same as that from $3 \times 10^4$ to $7 \times 10^4$ of (PSt)_{7.6}. Actually, the two $M_w/M_n$ values are less than 1.11, and the MWDs should be narrow. On the other hand, the $R_f$ values of the three samples defined by Eq. (9) were less than 20%. The DNAs in Fig. 5 might be concluded to be narrow.

The DNA of the (PSt)_{7.6} did not appear to shift to that of the (PSt)_{7.6}Star-(PIs)_{4.6} in parallel. However, a normalized DNA can be defined as plots of $w/f_{\text{max}}$ against $f/f_{\text{max}}$, where $w_{f_{\text{max}}}$ and $f_{\text{max}}$ are, respectively, a maximum of $w_f$ and that of $f$. The normalized DNA of the (PSt)_{7.6} was found to be the same as that of (PSt)_{7.6}Star-(PIs)_{4.6}. In other word, a half width of the normalized DNA of the (PSt)_{7.6} is the same as that of the (PSt)_{7.6}Star-(PIs)_{4.6}. Hence, the DNA of the (PSt)_{7.6} shifted to that of the (PSt)_{7.6}Star-(PIs)_{4.6} in parallel. Then, the DNA of the (PSt)_{7.6}Star-(PIs)_{4.6} also shifted to that of the (PSt)_{7.6}Star-(PIs)_{6.3} in parallel. These findings suggest that the anionic living polymerization of PIsm by the (PSt)_{7.6} and the (PSt)_{7.6}Star-(PIs)_{6.3} carbamions proceeded homogeneously. That is, a selective polymerization where PIsm is polymerized predominantly by the (PSt)_{7.6} and (PSt)_{7.6}Star-(PIs)_{6.3} having the lower $M_n$ did not occur under the present polymerization condition. The mechanism of anionic living polymerization of macromonomers for preparing hetero-star block copolymers from a DNA perspective will be discussed in detail elsewhere [14].

4. Conclusion

We have previously prepared (PSt)_{6.2}Star-PIs and (PSt)_{7.6}Star-(PIs)_{6.3} hetero-star block copolymers via anionic living polymerization of PSTm and PIsm macromonomers. By considering the composition distribution of the hetero-star block copolymers, the $M_i$ was precisely calculated to become the same equation as Eq. (3) using GPC/LALLS. Thus three-dimensional molecular weight distributions (3D-MWD) of the two (PSt)_{6.2}Star-(PIs)_{6.2}
and its four precursors were determined by plotting a weight fraction \((W_i^{\text{obs}})\) at the \(i\)th elution volume against the corresponding molecular weight \((M_i)\) and \(i\)th elution volume. A non-linear relationship between the \(i\)th elution volume and \(M_i\) of \((\text{PSt})_{n} \cdot \gamma\text{-star-PIs}\) was first observed and explained from a \(g\text{star}\) perspective.

The distributions of the number of arms (DNAs) of the two \((\text{PSt})_{f-n} \cdot \gamma\text{-star-PIs}\) and its four precursors were determined. The analysis for determining the DNA was evaluated as a reliability factor \((R)\) as defined as

\[
R = 100 \frac{\sum W_i^{\text{obs}} - W_i^{\text{cal}}}{\sum W_i^{\text{obs}}} \tag{10}
\]

The \(R\) values for the six samples were found to be less than 6\%. Hence, the DNAs shown in Fig. 4 and Fig. 5 might be concluded to be valid. The DNAs seem to be similar to the corresponding MWDs. Anionic living polymerization of \(\text{PIsM}\) by the \((\text{PSt})_{7.6} \cdot \gamma\text{-star-PIs}\) carbanions was found to proceed homogeneously from a DNA perspective.

**Appendix A**

The star block copolymers have a molecular weight distribution and a composition distribution (CD) among the different molecular weight and the same molecular weights. For this reason, we will consider the molecular weight, the refractive index increment and the concentration of the star block copolymer at the \(i\)th elution volume as \(M_{i,j}\), \((dn/dc)_{i,j}\), and \(C_{i,j}\), respectively. That is, a molecular weight distribution can be represented by the subscript \(i\), and the CD can be represented by the subscript \(j\). Thus the refractive index (RI) intensity \((H_i^{\text{RI}})\) and the LALLS intensity \((H_i^{\text{LS}})\) of the corresponding RI and LALLS chromatograms of the star block copolymers at the \(i\)th elution volume can be described as follows.

\[
H_i^{\text{RI}} = k_{\text{RI}} \sum_j (dn/dc)_{i,j} C_{i,j} \tag{A1}
\]

\[
H_i^{\text{LS}} = k_{\text{LS}} \sum_j [(dn/dc)_{i,j}]^2 M_{i,j} C_{i,j} \tag{A2}
\]

As a first approximation, one can assume that \(M_{i,j} = M_i\), because the polymer chains having \(M_{i,j}\) appeared at the same \(i\)th elution volume. As reported by the present authors elsewhere [15], the \(M_n^{\text{LALLS}}\) values determined using \(M_{i,j} = M_i\) were found to be equal to the \(M_n^{\text{OSM}}\) values determined by membrane osmometry for linear and star block copolymers.
The relation of $M_{i,j} = M_i$ is an assumption but was found to be used for determining $M_n^{\text{LALLS}}$ of the star block copolymers having sharp composition distributions in THF as a good solvent for PST and PIs. Therefore, $M_{i,j}$ in Eq. (A2) can be changed to $M_i$ and moved outside of the summation of $\Sigma_j$. By this treatment, Eq. (A3) can be derived as follows:

$$M_i = \frac{(k_{RI})^2 H_i^{LS} \left[ \sum_j (dn/dc)_{i,j} C_{i,j} \right]^2}{k_{LS} \left( H_i^{RI} \right)^2 \sum_j (dn/dc)_{i,j} C_{i,j}} \tag{A3}$$

With the relation of $\Sigma_j C_{i,j} = C_i$ in mind, the two summations in Eq. (A3) can be changed as follows: $\Sigma_j (dn/dc)_{i,j} C_{i,j} = C_i <(dn/dc)_{i,j}> = C_i (dn/dc)_i$ and $\Sigma_j [(dn/dc)_{i,j}]^2 C_{i,j} = C_i [(dn/dc)_i]^2$, where $< \ldots >$ means the average of the corresponding refractive index increments. The corresponding equations of $M_i$ thus became Eq. (3) in the text.
References

Figure Captions

Fig. 1  Three-dimensional molecular weight distributions (3D-MWDs) of the (PSt)$_{4.7}$, (PSt)$_{6.2}$, and (PSt)$_{6.2}$-star-PIs: Plots of $W_i^\text{obs}$ against $M_i$ and the $i$th elution volume.

Fig. 2  Three-dimensional molecular weight distributions (3D-MWDs) of the (PSt)$_{7.6}$, (PSt)$_{7.6}$-star-(PIs)$_{4.6}$, and (PSt)$_{7.6}$-star-(PIs)$_{6.3}$: Plots of $W_i^\text{obs}$ against $M_i$ and the $i$th elution volume.

Fig. 3  Each molecular weight distribution of the (PSt)$_f$ star polymers, [$M_{f,i}$~$W_{f,i}$], where $f$ is a number of arms ranging from 1 to 20.

Fig. 4  The GPC chromatograms, the molecular weight distributions (MWDs), and the distributions of the number of arms (DNAs) for (a) (PSt)$_{4.7}$, (b) (PSt)$_{6.2}$, and (c) (PSt)$_{6.2}$-star-PIs.

Fig. 5  The GPC chromatograms, the molecular weight distributions (MWDs), and the distributions of the number of arms (DNAs) for (a) (PSt)$_{7.6}$, (b) (PSt)$_{7.6}$-star-(PIs)$_{4.6}$, and (c) (PSt)$_{7.6}$-star-(PIs)$_{6.3}$. 

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Table 1. Molecular characteristics of the (PSt)$_{f,1}$-star-PIs$^a$ and (PSt)$_{f,n}$-star-(PIs)$_n^b$ hetero-star block copolymers.

<table>
<thead>
<tr>
<th>Polymers</th>
<th>10$^{-2}$M$_n$</th>
<th>M$_w$/M$_n$</th>
<th>Arme</th>
<th>w$^f$/%$^f$</th>
<th>$M_n$</th>
<th>ΣW$^f$</th>
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<tbody>
<tr>
<td></td>
<td>GPC$^c$</td>
<td>LALLS$^d$</td>
<td>GPC$^c$</td>
<td>LALLS$^d$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(PSt)$_{4.7}$</td>
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<td>2.58</td>
<td>1.16</td>
<td>1.15</td>
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<tr>
<td>(PSt)$_{6.2}$</td>
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<td>3.37</td>
<td>1.10</td>
<td>1.09</td>
<td>6.2</td>
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<tr>
<td>(PSt)$_{6.2}$-star-PIs</td>
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<td>1.05</td>
<td>1.04</td>
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<tr>
<td>(PSt)$_{7.6}$</td>
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<td>4.17</td>
<td>1.16</td>
<td>1.11</td>
<td>7.6</td>
<td></td>
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<tr>
<td>(PSt)$<em>{7.6}$-star-(PIs)$</em>{4.6}$</td>
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<td>6.61</td>
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<td>1.06</td>
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<tr>
<td>(PSt)$<em>{7.6}$-star-(PIs)$</em>{6.3}$</td>
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<td>1.07</td>
<td>1.06</td>
<td>13.9</td>
<td>45</td>
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</table>

$^a$ PStM macromonomer (M$_n$=5.45x10$^3$, M$_w$/M$_n$=1.03) was anionically polymerized by sec-BuLi in Bz at room temperature under a pressure of 10$^{-6}$ mmHg and isoprene was sequentially polymerized.

$^b$ PStM macromonomer was anionically polymerized by n-BuLi in THF at -78°C under a pressure of 10$^{-6}$ mmHg and the PI$^b$M macromonomer (M$_n$=5.33x10$^3$, M$_w$/M$_n$=1.04) was sequentially polymerized.

$^c$ Determined by GPC using standard PSts.

$^d$ A special analysis using GPC in conjugation with a low-angle laser light scattering detector.$^{15}$

$^e$ Average number of arms, <f> of the star polymers determined by <f>=M$_n$$_{star}$/M$_n$$_{arm}$, where M$_n$$_{star}$ and M$_n$$_{arm}$ are the molecular weights of the final star polymers and the corresponding macromonomers, respectively.

$^f$ PIs contents determined by M$_n$$_{PIs}$/M$_n$$_{star}$ and by GPC using UV and RI detectors.