Design of a quasi-optical mode conversion system with variable output beam size

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Design of a quasi-optical mode conversion system with variable output beam size

I. OGAWA, T. IDEHARA and W. KASPAREK

The quasi-optical system consisting of a quasi-optical antenna and two parabolic mirrors can convert the $\text{TE}_{01}$ mode output ($f = 301 \text{ GHz}$) of the Gyrotron FU IV into a well collimated, linearly polarized beam. This system offers the possibility of tuning the size of the beam produced from $5 \text{ mm}$ up to $24 \text{ mm}$ by rotating and moving the two parabolic mirrors.

1. Introduction

Electromagnetic waves in the submillimetre wavelength range are used for developments in numerous fields including material physics, plasma diagnostics, astronomy, biophysics and material processing. Some applications such as plasma scattering measurements (for example Fekete et al. 1994, Terumichi et al. 1984, Suvorov et al. 1997) need more intense waves.

Molecular vapour lasers (Semet et al. 1980, Kawahata et al. 1988) and backward-wave oscillators (Morino et al. 1997) have been used as a power source in this wavelength range because they produce stable continuous wave (CW) outputs as Gaussian beams. On the other hand, high frequency gyrotrons are the most promising candidates for delivering intense waves of several hundred watts up to several kilowatts (Zaytsev et al. 1974, Spira-Hakkarainen et al. 1990, Idehara et al. 1995). In addition, CW operation has been achieved with high frequency gyrotrons (Hong et al. 1993, Idehara et al. 1998).

The gyrotron output should be converted into a well collimated, linearly polarized beam because the gyrotron delivers the $\text{TE}_{nm}$ waveguide mode output, which is far from what is usually required as radiation power source.

Some applications of gyrotrons such as the effective coupling of beams to hybrid modes in corrugated waveguides (for example, Ohkubo et al. 1997) require matching of the size of the beam produced.

A quasi-optical antenna (Vlasov and Orlova 1974) converts the gyrotron output into a linearly polarized beam whose far-field consists of a main beam with an elliptical cross-section and additional sidelobes. As can be seen from Gaussian optics, the main beam can be converted into a circular beam by two-dimensionally adjusting the spotsize and the curvature radius of the wave front of the beam. This conversion is attained by the combination of a parabolic cylinder and a modified
mirror similar to an ellipse or the combination of two parabolic cylinders (Ogawa et al. 1999a). The former is suitable for getting a high quality beam, while the latter is suitable for adjusting the beam waist size because a parabolic cylinder offers the tunability of focal lengths via the variation of the angle of incidence.

As an example of this approach, the design of the quasi-optical system for the CW TE_{03} mode output ($f = 301$ GHz) of the Gyrotron FU IV (Idehara et al. 1998) is presented.

2. Treatment of the beam by using Gaussian optics

The first element in the quasi-optical transmission line is the quasi-optical antenna (figure 1) consisting of a circular waveguide (internal radius $a_w = 14$ mm) with a step-cut and a cylindrical parabolic reflector (focal length $f_p = 21.75$ mm). The antenna converts the gyrotron output (TE_{03} mode, $f = 301$ GHz, Brillouin angle $\alpha = 6.62^\circ$) into a linearly polarized beam. The electric and magnetic fields are parallel to the x- and y-directions, respectively.

Radiation reflected from the parabolic reflector of the quasi-optical antenna is treated as if it came from a plane image source lying behind the reflector (Wada and Nakajima 1986, Brand et al. 1990).

In the design of the system (figure 2), the image source is located thus that the beam with polarization in the x-direction propagates along the z-axis. The profiles of the beam produced by the image source are calculated using the Huygens equation (Ogawa et al. 1997).

Mirror m1 focuses the beam in the y'-direction and mirror m2 focuses the beam in the x''-direction. The beam produced at mirror m1 contains sidelobes in addition to the main lobe, as can be seen in figure 3. In order to truncate the sidelobes, the size of the rectangular mirror m1 is chosen to be 179 mm in the x-direction and 240 mm in the y-direction. This is possible, as for the application planned the transmission efficiency is of minor importance, whereas beam quality is a strong requirement. On the other hand, mirror m2 is wide enough to avoid any diffraction loss due to beam truncation.

The distance between the image source and the mirror m1 is large (7000 mm) enough to guarantee that the spot sizes of the main beam are accurately given by assuming a bi-Gaussian beam whose waist ($w_{0x} = 38.1$ mm in the x-direction, $w_{0y} = 25.3$ mm in the y-direction) is located at the centre of the image source.

![Figure 1. Quasi-optical antenna. The x- and y-directions correspond to the directions of electric and magnetic fields, respectively.](image-url)
Figure 2. Quasi-optical system. (a) A plane image source is used for the calculation of the subsequent radiation patterns of the quasi-optical antenna. The mirror m1 focuses the beam in the $y'$-direction. (b) The mirror m2 focuses the beam in the $x''$-direction.

Figure 3. Calculated intensity contours at the mirror m1. Contours are in decibels relative to the intensity maximum.
The intensity of the bi-Gaussian beam is given by

$$I = \frac{2P_0}{\pi w_x w_y} \exp \left( -\frac{2x^2}{w_x^2} \right) \exp \left( -\frac{2y^2}{w_y^2} \right)$$  \hspace{1cm} (1)$$

where $w_x$ and $w_y$ are the spot sizes of the bi-Gaussian beam in the $x$- and $y$-directions, respectively and $P_0$ is the total beam power.

The complex beam parameters $q_x$ and $q_y$ in the $x$- and $y$-directions defined by

$$\begin{align*}
\frac{1}{q_x} &= \frac{1}{R_x} - j \frac{\lambda}{\pi w_x^2} \\
\frac{1}{q_y} &= \frac{1}{R_y} - j \frac{\lambda}{\pi w_y^2}
\end{align*}$$  \hspace{1cm} (2)$$

are convenient parameters to treat the beam propagation and the focusing due to the focusing element (Ogawa et al. 1999 b), where $R_x$ and $R_y$ are the curvature radii of the wave fronts in the $x$- and $y$-directions, respectively.

At the beam waist, $R_x, R_y \to \infty$. Therefore, in this case the complex beam parameters are

$$q_{0x} = j \frac{\pi w_0^2}{\lambda}$$
$$q_{0y} = j \frac{\pi w_0^2}{\lambda}$$  \hspace{1cm} (3)$$

The complex beam parameters change, when the beam propagates or it is focused by any element. The complex beam parameters $q_x$ and $q_y$ change to

$$\begin{align*}
q'_x &= q_x + d \\
q'_y &= q_y + d
\end{align*}$$  \hspace{1cm} (4)$$

after propagating a distance $d$.

After a focusing element, they change to new values given by

$$\begin{align*}
\frac{1}{q'_x} &= \frac{1}{q_x} - \frac{1}{f_x} \\
\frac{1}{q'_y} &= \frac{1}{q_y} - \frac{1}{f_y}
\end{align*}$$  \hspace{1cm} (5)$$

where $f_x$ and $f_y$ are the focal lengths of the focusing element in both directions.

3. Design of the system using Gaussian optics

To demonstrate the method, we have made a design of the system (figure 2) to produce a well collimated beam with a circular cross-section (waist size of $w_{0x'} = w_{0y'} = w_0'$).

The design is such that it converts the bi-Gaussian beam with complex beam parameters $q_{0x}$ and $q_{0y}$ at the image source to that with complex beam parameters $q_{0x'}$ and $q_{0y'}$ at the beam waist according to equations (4) and (5). The values of $q_{0x}$ and $q_{0y}$ are obtained using equation (3) with $w_{0x} = 38.1 \text{ mm}$ and $w_{0y} = 25.3 \text{ mm}$, respectively. The values of $q_{0x'}$ and $q_{0y'}$ are also obtained using equation (3) with $w_{0x'} = w_{0y'} = w_0'$, respectively.

As can be seen from equation (2), we need two procedures to equalize the two-dimensionally different complex parameters ($q_{0x} \neq q_{0y}$) into the same value.
(q_{0x'} = q_{0y'})$, namely, the coincidence of the spot sizes and that of the curvature radii of the wave fronts. As can be seen from equations (4) and (5), the former consists of focusing and propagation, the latter consists of focusing only.

The mirror m1 plays the role of equalizing both spot sizes at the mirror m2. As can be seen from equation (2), this is expressed by

$$\text{Im} \left( \frac{1}{q_{2x'}} \right) = \text{Im} \left( \frac{1}{q_{2y'}} \right)$$  \hspace{1cm} (6)

where $q_{2x'}$ and $q_{2y'}$ are the complex beam parameters in the $x'$- and $y'$-directions at the mirror m2. If the focal length $f_{1y}$ of mirror m1 is given, the distance $d_{12}$ between the mirrors m1 and m2 is obtained by equation (6).

The value of $f_{1y}$ can be determined by the condition that the beam focused by the mirror m1 has a waist size of $w_{0y}$. As can be seen from equations (4) and (5), they are given by

$$\frac{1}{q_{1y'}} = \frac{1}{q_{1y}} - \frac{1}{f_{1y}}$$  \hspace{1cm} (7)

where $q_{1y}$ and $q_{1y'}$ are the complex beam parameters at mirror m1 and just after reflection by mirror m1, respectively and $d'$ is the propagation distance from the mirror m1 to the beam waist. If we notice that $q_{0y'}$ is purely imaginary, $d'$ is given by

$$d' = -\text{Im}(q_{y'})$$  \hspace{1cm} (8)

Then, the distance $d''$ between the mirror m2 and the beam waist is given by

$$d'' = d' - d_{12}$$  \hspace{1cm} (9)

Although the beam has a circular cross-section at the mirror m2, the curvature radius of the wave front in the $x''$-direction is different from that in the $y''$-direction. The mirror m2 plays the role of equalizing both the curvature radii of the wave fronts. This is expressed by

$$\frac{1}{q_{2x''}} = \frac{1}{q_{2x}} - \frac{1}{f_{2x}}$$  \hspace{1cm} (10)

where $f_{2x}$ is the focal length of mirror m2, and $q_{2x'}$ and $q_{2y'}$ are the complex beam parameters at the mirror m2 and $q_{2x'}$ is that just after reflection by the mirror m2.

Gaussian optics has been applied to the system, which produces beams with different waist sizes. Three cases are presented: Case I, $w_0'' = 4.0$ mm; Case II, $w_0'' = 10.0$ mm; and Case III, $w_0'' = 20.0$ mm. The results obtained by Gaussian optics are listed in table 1.

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<th>Mirror m1</th>
<th>Mirror m2</th>
<th>Beam waist</th>
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<tr>
<td>$f_{1y}$ (mm)</td>
<td>$w_x$ (mm)</td>
<td>$w_y$ (mm)</td>
</tr>
<tr>
<td>Case I</td>
<td>1001</td>
<td>69.6</td>
</tr>
<tr>
<td>Case II</td>
<td>2096</td>
<td>69.6</td>
</tr>
<tr>
<td>Case III</td>
<td>3319</td>
<td>69.6</td>
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Table 1. The results obtained by Gaussian optics. $w_x$ and $w_y$ are the spot sizes in the $x$- and $y$-directions, respectively. $w_{0x'}$ and $w_{0y'}$ are the waist sizes in the $x''$- and $y''$-directions, respectively.
4. Mirrors m₁ and m₂

The mirror m₁ is a parabolic cylinder whose focal axis is located in the \( x'-z' \) plane (figure 4). If the rays parallel (opposite) to the \( x' \)-axis are incident on the mirror at an angle of \( \theta_y \), the rays reflected by the mirror m₁ intersect at the focal point \( F_1 \). In the present case, the mirror m₁ functions as a focusing element with the focal length of

\[
f_{1y} = \frac{f_y}{\cos \theta_y}
\]

(11)

where \( f_y = 707.6 \text{ mm} \) is the focal length of the parabolic cylinder.

The mirror m₂ is also a parabolic cylinder whose focal axis is located in the \( y''-z'' \) plane (figure 5). The mirror m₂ functions as a focusing element with the focal length of

\[
f_{2x} = \frac{f_x}{\cos \theta_x}
\]

(12)

where \( f_x = 585.5 \text{ mm} \) is the focal length of the parabolic cylinder.

The angles \( \theta' \) and \( \theta'' \) in figure 2 are related to those \( \theta_y \) and \( \theta_x \), respectively. The relations are given by

\[
\begin{align*}
\theta' &= \pi - 2\theta_y \\
\theta'' &= \pi - 2\theta_x
\end{align*}
\]

(13)

Figure 4. Geometry of the mirror m₁. The mirror is a parabolic cylinder with a focal length of \( f_y \). Its focal axis is in the \( x'-z' \) plane.

Figure 5. Geometry of the mirror m₂. The mirror is a parabolic cylinder with the focal length of \( f_x \). Its focal axis is in the \( y''-z'' \) plane.
If necessary, problems due to changes of the system geometry can be avoided by introducing plane mirrors.

5. Verification by the calculation using the Huygens equation

In order to verify the obtained results using Gaussian optics, we have carried out numerical calculations of the Huygens equation. At first, the incident electromagnetic fields at the surface of the mirror m1 are calculated. The electromagnetic fields reflected by the mirror are obtained using the boundary conditions for a perfect conductor. The electromagnetic fields on the subsequent mirror m2 are obtained by repeatedly using the Huygens equation and the boundary condition together with the previous calculated results as the sources.

For Case I ($w_0'' = 4.0\,\text{mm}$), calculated intensity contours are shown in figure 6. The main beam with elliptical cross-section at mirror m1 (figure 3) approaches to

![Figure 6](image)

Figure 6. Calculated intensity contours corresponding to the Case I ($w_0'' = 4.0\,\text{mm}$) (a) at the mirror m2, (b) at the beam waist predicted by Gaussian optics. Contours are in decibels relative to the intensity maximum. (c) and (d) at the vicinity of the beam waist predicted by Gaussian optics. Contours are relative to the intensity along the $z''$-axis.
almost circular at mirror m2 (figure 6(a)) for the sake of the mirror m1 and is well-
collimated at the beam waist as is predicted by Gaussian optics (figure 6(b)). But, the
beam has a triangular cross-section.

For Case II ($w_0'' = 10.0\text{ mm}$) and Case III ($w_0'' = 20.0\text{ mm}$), calculated intensity
contours are shown in figure 7 and figure 8, respectively. As the beam waist size is
increased, the beam shape approaches to circular.

The beam produced by the image source contains sidelobes in addition to the
main beam, as can be seen in figure 3. The quality of the beam is improved by
truncating the side lobes by limiting the size of the mirror m1 to the optimum size.
This attempt is effective in removing sidelobes. In spite of the truncation of the
sidelobes, most of power from the image source (82.6%) is still reflected by the
mirror m1.

Arrangements of the mirrors and results obtained by the calculations are listed in
table 2. The results obtained by the Gaussian optics approach (table 1) are in good
agreement with the calculations.

![Figure 7](image)

Figure 7. Calculated intensity contours corresponding to the Case II ($w_0'' = 10.0\text{ mm}$) (a) at
the mirror m2, (b) at the beam waist predicted by Gaussian optics. Contours are in
decibels relative to the intensity maximum. (c) and (d) at the vicinity of the beam waist
predicted by Gaussian optics. Contours are relative to the intensity along the $z''$-axis.
Figure 8. Calculated intensity contours corresponding to the Case III ($w_0'' = 20.0\text{mm}$) (a) at the mirror m2, (b) at the beam waist predicted by Gaussian optics. Contours are in decibels relative to the intensity maximum. (c) and (d) at the vicinity of the beam waist predicted by Gaussian optics. Contours are relative to the intensity along the $z''$-axis.

<table>
<thead>
<tr>
<th>Mirror m1 $f_s = 707.6 \text{ (mm)}$</th>
<th>Mirror m2 $f_s = 585.5 \text{ (mm)}$</th>
<th>Beam waist</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_y$ $w_x$ $w_y$ (mm) $d_{12}$ $\theta_z$ $w_{x'}$ $w_{y'}$ (mm) $d_{x''}$ $d_{y''}$ $w_{x''}$ $w_{y''}$ (mm)</td>
<td>$\theta_y$ $w_x$ $w_y$ (mm) $d_{12}$ $\theta_z$ $w_{x'}$ $w_{y'}$ (mm) $d_{x''}$ $d_{y''}$ $w_{x''}$ $w_{y''}$ (mm)</td>
<td>$\theta_y$ $w_x$ $w_y$ (mm) $d_{12}$ $\theta_z$ $w_{x'}$ $w_{y'}$ (mm) $d_{x''}$ $d_{y''}$ $w_{x''}$ $w_{y''}$ (mm)</td>
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<td>Case I 45.0 69.6 91.4 251.9 45.0 63 69 891 899 5.0 5.1</td>
<td>Case II 70.3 69.6 91.4 563.5 72.1 67 73 2751 2788 12.2 11.9</td>
<td>Case III 77.7 69.6 91.4 965.0 80.1 72 76 5029 5211 23.7 23.3</td>
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</table>

Table 2. Arrangement of the mirrors and results obtained by calculations using the Huygens equation. $w_x$ and $w_y$ are the spot sizes in the x- and y-directions, respectively. $d_{x''}$ and $d_{y''}$ are the distances between the mirror m2 and the waist, respectively.
6. Conclusion

A system consisting of a quasi-optical antenna and two parabolic mirrors can convert the TE_{01} mode output (f = 301 GHz) of the Gyrotron FU IV into a well collimated, linearly polarized beam. This system offers the function to tune the size of the beam produced from 5 mm up to 24 mm by rotating and moving the two parabolic mirrors. The arrangement of the mirrors are obtained by using Gaussian optics.

For the sake of the beam quality, sidelobes of the beam produced by the quasi-optical antenna are removed by locating the first mirror of suitable size far from the antenna. In spite of the truncation of the sidelobes, the beam produced by the system maintains most of power from the image source (~ 80%).

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Numerical calculations were made at the National Institute for Fusion Science Computer Center.

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