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Maximum Stress Intensity Factor for a Circumferential Crack in a Finite-Length Thin-Walled Cylinder under Transient Radial Temperature Distribution

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ABSTRACT

Transient stress intensity factor of an axisymmetric circumferential crack in a finite-length, thin-walled and edges rotation-restrained cylinder, which is suddenly cooled inside from uniform temperature distribution was formulated. Then the effects of various structural parameters and heat transfer conditions on the value were studied. The maximum transient stress intensity factor in a thermal cycle showed a tendency to decrease monotonously when crack length was varied to become longer than a specific value. Furthermore, the maximum transient stress intensity factor for an infinite cylinder length was smaller than that for a specific length when cylinder length was varied.

Key Words: fracture mechanics, stress intensity factor, thermal Stress, thin-walled cylinder, circumferential crack, crack arrest

1. INTRODUCTION

Crack propagation occurs, in general, under cyclic thermal loads as well as under cyclic external loads. However, in the absence of additional cyclic external loads, the crack propagation may not always lead to a through-thickness fracture under cyclic thermal loads. For example, there are interesting experimental data which indicate a surface axisymmetric circumferential crack inside a hollow cylinder shows tendency toward crack arrest, when the inside of the cylinder is cyclically cooled from uniform temperature distribution [1]. If the crack propagation rate in this situation fits the Paris law [2], this tendency may be explained by investigating the characteristics of the stress intensity factor (SIF) range for each cycle. As the SIF for uniform temperature distribution is zero [3], SIF range in interest is the maximum transient SIF itself. This SIF for the problem is affected by various factors such as cylinder configuration, edge restraint and cooling rate, etc. So the effects of these factors on the SIF should be systematically evaluated to grasp the characteristics and to understand the crack arrest tendency through them. The SIF can be investigated numerically by FEM, or analytically by assuming long cylinder and radial temperature distribution [4]. But, these methods are not realistic, taking the required efforts into account, to evaluate the SIF systematically considering the various factors above and to grasp the characteristics. An alternative is desired.

Keeping these situations in mind, as a first step, we derived a closed form equation of the SIF for an axisymmetric circumferential crack in a finite-length thin-walled cylinder with its edges rotation-restrained and subjected to a given linear radial temperature distribution [3] (*SIF under linear temperature distribution*). As linearization of a given radial temperature distribution is often applied in approximate evaluation, the characteristics of the SIF for this linear temperature distribution was thought to represent some characteristics of the maximum transient SIF in interest. Advantage of our closed form SIF was that it can evaluate the effects of various structural parameters, even crack location and cylinder length. The characteristics of the *SIF under linear temperature distribution* was then studied by varying structural parameters and the following results were obtained [3]: (1) The SIF shows its maximum when the crack is located at the midpoint of the cylinder length. (2) The SIF is strongly affected by the cylinder length. (3) The SIF decreases monotonously when the crack length becomes lager than a specific value (*tendency of the SIF to decrease for long crack*). This *tendency of the SIF to decrease for long crack* corresponds to the crack arrest tendency.

As the *SIF under linear temperature distribution* showed a characteristic corresponding to the crack arrest tendency, we were encouraged to proceed to study the characteristics of the maximum transient SIF of the crack in the identical cylinder structure, which is suddenly cooled inside from uniform temperature distribution and adiabatically insulated outside. In the past, Nied and Erdogan [4] analytically evaluated the transient SIF of the crack for a condition of an infinitely long cylinder and an infinite heat transfer coefficient. However, their solution does not refer to the effect of cylinder length on the SIF. On the other hand, the test specimens used in experiments [1] are too short to satisfy the long cylinder assumption, and we know from the fact (3) above that the effect of cylinder length on the SIF should be considered. So we will improve our closed form equation of the *SIF under linear temperature distribution* so that it can treat transient (non-linear) temperature distribution in this paper. In addition, we will study the characteristics of the maximum transient SIF (important in structural integrity evaluation) for various heat transfer conditions as well as various structural parameters such as cylinder length.

In the following, the SIF of an axisymmetric circumferential crack in a finite-length thin-walled cylinder with its edges rotation-restrained, subjected to a given non-linear radial temperature distribution will be formulated first. The weight function of the crack derived by the authors [5] is combined with the procedure applied in deriving the *SIF under linear temperature distribution* [3] in formulation. Note that our weight function was newly developed to include all the structural parameters necessary, even the cylinder length. It is assumed that the crack is located at the midpoint of the cylinder length considering the fact (1) above.

Subsequently, analytical solution for the transient radial temperature distribution of the uncracked long hollow cylinder, which is adiabatically insulated on the outer surface and suddenly cooled inside from uniform temperature distribution, will be introduced.

By combining these two, the effects of structural parameters and heat transfer condition on the maximum transient SIF under this transient temperature distribution are studied. The results obtained are as follows: i) The maximum transient SIF for the problem also shows a *tendency to decreases for long crack*, regardless of the heat transfer condition. ii) The SIF is strongly affected by the cylinder length. iii) The SIF for a specific cylinder length is larger than that for an infinite length. Considering these results, the tendency toward crack arrest (when this thermal stress cycle is repeated) can be justified, assuming Paris law.

2. SIF OF AN AXISYMMETRIC CIRCUMFERENTIAL CRACK IN A FINITE-LENGTH THIN-WALLED CYLINDER UNDER TRANSIENT RADIAL TEMPERATURE DISTRIBUTION

2. 1 SIF of an axisymmetric circumferential crack in a finite-length thin-walled cylinder under non-linear radial temperature distribution

The SIF of a circumferential crack in a finite-length thin-walled cylinder, which is subjected to non-linear radial temperature distribution $T(\eta)$ (shown in Figure 1), is formulated first. The crack is located at the midpoint of the cylinder length H and the edges of the cylinder are rotation-restrained. The edges can move freely in the axial direction, as encountered in practical problems. The material of the cylinder is assumed to be homogeneous with isotropic and temperature independent physical properties. Bernoulli-Euler assumption that sections which are plane and perpendicular to the axis before loading remains so after loading is applied. In the following, we will modify the procedure adopted in obtaining the *SIF under linear temperature distribution* [3] to obtain the desired SIF under non-linear temperature distribution.

First, based on superposition principle, the desired SIF K_{cyl} of Figure 2 (i) is obtained as the sum of K_{free} in Figure 2 (ii) (SIF due to free expansion) and K_{r} in Figure 2 (iii) (SIF due to edge restraint):

$$K_{\text{cyl}}=K_{\text{free}}+K_{\text{r}} \tag{1}$$

Here, ($-M_{\text{r1}}$) and ($-M_{\text{r2}}$) are the bending loads to cancel the inclination angles due to free expansion θ_{t1} and θ_{t2}, respectively. Note that the crack for the problem in this paper is located at the midpoint of the cylinder length ($h_1=h_2=H/2$), whereas the geometry in Figures 2 and 3 represents the general case of arbitrarily crack location.

Second, based on the Duhamel's analogy (used to replace thermoelastic problems with isothermal problems) [6], the deformation due to free expansion (Figure 3 (a)) is obtained as the sum of deformations in Figure 3 (b), (c), (d) and (e). σ_M and σ_P in Figure 3 are defined as follows:

$$\sigma_M=\frac{E\alpha}{1-\nu}\left(T(\eta)-T_{\text{avg}}\right) \tag{2}$$

$$\sigma_P = \frac{E\alpha}{1-\nu} T_{\mathrm{avg}} \tag{3}$$

where T_{avg}, E, α and ν are the average temperature, Young's modulus, coefficient of thermal expansion and Poisson's ratio, respectively. As the SIFs due to the uniformly distributed force σ_P in Figure 3 (d) and (e) compensate each other, the sum of the SIFs for Figure 3 (b) and (c) gives the desired SIF for Figure 3 (a).

In case of linear $T(\eta)$ distribution, the stress distribution σ_M in Figure 3 (b) and ($-\sigma_M$) in Figure 3 (c) can be replaced with the axisymmetric bending loads M_t applied to the cylinder edges and the linear stress distribution corresponding to the resultant moment ($-M_t$), respectively [3]. Here, M_t was the bending moment corresponding to the linear stress distribution σ_M and given by

$$M_t = \int_{-W/2}^{+W/2} \sigma_M \cdot \eta d\eta = \frac{E\alpha}{1-\nu} \int_{-W/2}^{+W/2} T(\eta) \cdot \eta d\eta \tag{4}$$

M_t was named as *equivalent thermal moment* [3].

In case in interest, in which $T(\eta)$ is non-linear, situation is not the same. Although the replacement of the non-linear stress distribution σ_M by M_t means to approximate σ_M distribution as a linear one, St. Venant's principle (which describes that the effect of non-linearity in prescribed stress on the edges disappears quickly in the longitudinal direction) supports this approximation in case of obtaining the SIF $K_{\mathrm{free\,b}}$ for Figure 3 (b). However, it is necessary to appropriately include the effect of non-linearity in the stress distribution σ_M, in evaluating the SIF $K_{\mathrm{free\,c}}$ for Figure 3 (c).

So, we will apply the closed form equation to evaluate the SIF of the crack with the cylinder ends loaded by a pair of bending loads [7] to evaluate $K_{\mathrm{free\,b}}$ as in Ref. [3]. By substituting $M=M_t$ and $h_1=h_2=H/2$ to Eqs. (9) and (10) in Ref. [7], $K_{\mathrm{free\,b}}$ is deduced as follows:

$$K_{\mathrm{freeb}} = \left[\frac{2(\sinh\frac{\beta H}{2}\cos\frac{\beta H}{2} + \sin\frac{\beta H}{2}\cos\frac{\beta H}{2})}{\sinh\beta H + \sin\beta H + \beta D \cdot \Delta\lambda(\xi) \cdot (\cosh\beta H + \cos\beta H - 2)} \right] \cdot \left[\frac{M_t}{Z}\sqrt{\pi a} \cdot F_M(\xi) \right] \tag{5}$$

Here, the term in [ ] is a closed form equation to evaluate the SIF of an edge-cracked strip under pure bending M_t, $Z = W^2/6$ is the section modulus and $F_M(\xi = a/W)$ is the infinite length edge-cracked beam's correction factor for finite width under pure bending. Furthermore, $\Delta\lambda$ is the increment of compliance due to the presence of a crack for an infinitely long beam under pure bending, β and D are quantities used in replacing cylindrical shell by a beam on an elastic foundation given concretely in the APPENDIX.

On the other hand, we will apply our weight function $w(x; a)$ [5] to evaluate the SIF of the crack subjected to arbitrarily distributed stress ($-\sigma_M$) on its surface to evaluate $K_{\mathrm{free\,c}}$ as:

$$K_{\mathrm{freec}}=\int_0^a \frac{R_{\mathrm{i}}+x}{R_{\mathrm{i}}+a}\left[-\sigma_M\right]\cdot w(x;a)dx \quad (6)$$

Note that our weight function was derived to include the effect of cylinder length on the SIF. The form of our weight function is given in APPENDIX.

The remaining SIF K_{r} is related to the edge rotation far away from the crack tip, thus it can be evaluated as the SIF of the crack in a cylinder with ends loaded by equivalent moment ($-M_{\mathrm{r1}}$) and ($-M_{\mathrm{r2}}$) in Figure 2. By substituting $h_1 = h_2 = H/2$ to Eqs. (7) through (10) in Ref. [3], K_{r} is deduced as follows:

$$K_{\mathrm{r}}=\left[\frac{1}{1+\dfrac{\beta D\cdot\Delta\lambda(\xi)\cdot(\sinh\beta H+\sin\beta H)}{\cosh\beta H-\cos\beta H}}-\frac{\sinh\beta H+\sin\beta H-2(\sinh\frac{\beta H}{2}\cos\frac{\beta H}{2}+\sin\frac{\beta H}{2}\cosh\frac{\beta H}{2})}{\sinh\beta H+\sin\beta H+\beta D\cdot\Delta\lambda(\xi)\cdot(\cosh\beta H+\cos\beta H-2)}\right]\cdot\left[\frac{(-M_t)}{Z}\sqrt{\pi a}F_M(\xi)\right] \quad (7)$$

For the ease of actual calculation, the desired SIF K_{cyl} in Eq. (1) is deduced further, by combining Eq. (5) and (7).

$$K_{cyl}=K_{freec}+K_{freeb}+K_r\equiv K_{freec}+K_{fbr} \quad (8)$$

$$K_{fbr}\equiv F_{t\,fbr}\cdot\left\{\frac{(-M_t)}{Z}\sqrt{\pi a}\cdot F_M(\xi)\right\} \quad (9)$$

$$F_{t\,fbr}=-2\beta D\cdot\Delta\lambda(\xi)\cdot\left[\cosh\beta H-\cos\beta H+\sinh\beta H\sin\beta H\right]/ \{[\sinh\beta H+\sin\beta H+\beta D\cdot\Delta\lambda(\xi)\cdot(\cosh\beta H+\cos\beta H-2)]\times [\cosh\beta H-\cos\beta H+\beta D\cdot\Delta\lambda(\xi)\cdot(\sinh\beta H+\sin\beta H)]\} \quad (10)$$

Thus, the desired SIF K_{cyl} is obtained as a sum of $K_{\mathrm{free\,c}}$ (obtained by Eq. (6)) and K_{fbr} (obtained by Eqs. (9) and (10)).

As we will compare the SIF for non-linear and linear temperature distribution later, let's define the *SIF under linear temperature distribution* K_{tcyl} for the configuration in Figure 1. By substituting $h_1 = h_2 = H/2$ to eqs. (5) through (11) in Ref. [3] and by introducing function F_{tcyl}, K_{tcyl} is deduced as follows:

$$K_{\mathrm{tcyl}}=F_{\mathrm{tcyl}}\cdot\left[\frac{(-M_{\mathrm{t}})}{Z}\sqrt{\pi a}\cdot F_M(\xi)\right] \quad (11)$$

$$F_{\mathrm{tcyl}}=\frac{\cosh\beta H-\cos\beta H}{(\cosh\beta H-\cos\beta H)+\beta D\cdot\Delta\lambda(\xi)\cdot(\sinh\beta H+\sin\beta H)} \quad (12)$$

2. 2 Analytical solution for the transient radial temperature distribution and the transient SIF

The transient thermal stress problem considered here is that of an infinitely long hollow cylinder, which is adiabatically insulated on the outer surface and is suddenly cooled inside from uniform temperature 0 K at time $\tau = 0$ sec. The coefficient of thermal conductivity, inner and outer radii of the cylinder are Λ W/(m·K), r_{i} and r_{o} m, respectively. The coolant temperature is ($-2\Delta T$) K and the heat transfer coefficient between coolant and wall is h W/(m^2·K). The analytical solution for the transient radial temperature $T(\tau, r)$ for location r m and time τ sec of the problem has been given by Koizumi et al. [8] as follows:

$$T(\tau,r) = -2\Delta T\left[1-\sum_{n=1}^{\infty}\exp(-\kappa d_n^2\tau)\frac{\pi c_n^2\chi\cdot F_0\,(d_n r)}{c_n^2(d_n^2+\chi^2)-a_n^2 d_n^2}\right] \quad \textbf{(13)}$$

where the coefficients in this equation are given as follows, by using constants $\chi = h/\Lambda$ m^{-1} and κ m^2/s : coefficient of diffusion.

$$\left.\begin{aligned} &F_0(d_n r)=b_n J_0(d_n r)-a_n Y_0(d_n r)\\ &a_n=d_n J_1(d_n r_\mathrm{i})+\chi J_0(d_n r_\mathrm{i})\\ &b_n=d_n Y_1(d_n r_\mathrm{i})+\chi Y_0(d_n r_\mathrm{i})\\ &c_n=d_n J_1(d_n r_\mathrm{o})\\ &e_n=d_n Y_1(d_n r_\mathrm{o})\end{aligned}\right\} \quad \textbf{(14)}$$

J_0 and J_1 in Eq. (14) are the Bessel functions of the first kind of order 0 and 1, respectively, and Y_0 are Y_1 are the Bessel functions of the second kind of order 0 and 1, respectively. The coefficient d_n represents the positive root of the following equation.

$$a_n e_n - b_n c_n = 0 \quad \textbf{(15)}$$

Note that $T(\tau, r)$ is proportional to the initial temperature difference between the coolant and the cylinder ($-2\Delta T$).

As $T(\tau, r)$ for the problem was formulated, let's obtain the desired transient SIF K_{cyl}, corresponding to this $T(\tau, r)$. By substituting $T(\tau, r)$ given by Eq. (13) to Eqs. (2) and (4), the transient stress distribution σ_M used in evaluating the transient SIF $K_{\mathrm{free\ c}}$ by Eq. (6) and the transient M_t used in evaluating the transient K_{fbr} by Eq. (9) are obtained, respectively. Finally, the desired transient SIF K_{cyl} is obtained as the sum of $K_{\mathrm{free\ c}}$ and K_{fbr}, as shown in Eq. (8).

3. MAXIMUM SIF FOR AN AXISYMMETRIC CIRCUMFERENTIAL CRACK IN A FINITE-LENGTH THIN-WALLED CYLINDER UNDER TRANSIENT RADIAL TEMPERATURE DISTRIBUTION

3. 1 Comparison of the derived SIF with an analytical one

In this section, the transient temperature distribution $T(\tau, r)$ of an infinitely long hollow cylinder, which is adiabatically insulated outside and suddenly cooled inside from uniform temperature 0 K at time $\tau = 0$ sec was calculated by Eq. (13) first. The temperature of the coolant and heat transfer coefficient used for this study was ($-2\Delta T$) = -100 K and $h = \infty$ W/(m^2·K), respectively. After that, the transient SIF was calculated by Eq. (8) and compared with the analytical solution by Nied and Erdogan [4], to show the validity of the equation.

In this case, $T(\tau, r)$ is obtained as follows by putting $h \to \infty$ in Eqs. (13) and (14),

$$\frac{T(\tau,r)}{(-2\Delta T)}=1-\sum_{n=1}^{\infty}\exp(-\kappa d_n^2\tau)\frac{\pi\left[Y_0(d_n r_{\rm i})J_0(d_n r)-J_0(d_n r_{\rm i})Y_0(d_n r)\right]}{1-\left[J_0(d_n r_{\rm i})/J_1(d_n r_{\rm o})\right]^2} \quad (16)$$

where the coefficient d_n represents the positive root of the following equation.

$$Y_0(d_n r_{\rm i})Y_1(d_n r_{\rm o})-Y_0(d_n r_{\rm i})J_1(d_n r_{\rm o})=0 \quad (17)$$

Note that the time τ for this $T(\tau, r)$ can be represented through the dimensionless Fourier No. = $\kappa\tau/r_{\rm o}^2$ (or $\kappa\tau/r_{\rm i}^2$), as d_n is determined by inner and outer radii $r_{\rm i}$ and $r_{\rm o}$ in Eq. (17).

As for the cylinder configuration used in this numerical example, $r_{\rm i}$ = 90, $r_{\rm o}$ = 100 mm (W = 10 mm and $R_{\rm m}/W$ = 9.5) and the crack length a = 5 mm (a/W = 0.5). Cylinder length $H = 5/\beta$ = 126 mm was chosen to meet the condition of a long cylinder given by Labbens et al. [9]. Material constants E = 198 GPa, α = 16 x 10^{-6} /K, ν = 0.3, Λ = 12.7 W/(m·K) and κ = 3.61 mm^2/s were chosen, assuming austenitic steel.

For the case under consideration, $T(\tau, r)$ was calculated by Eq. (16) and shown in Figure 4. $T(\tau, r)$ is shown as a non-dimensional temperature $T/(2\Delta T)$, as it is proportional to $2\Delta T$. Subsequently, the transient *equivalent thermal moment* ($-M_{\rm t}$) calculated by Eq. (4) for this temperature distribution is shown in Figure 5 as a non-dimensional value normalized by $(-M_{\rm t})_{\Delta T}$ given by the following equation:

$$(-M_{\rm t})_{\Delta T}=\frac{E\alpha\Delta T}{1-\nu}\frac{W^2}{6} \quad (18)$$

which is the ($-M_{\rm t}$) for a linear temperature distribution with its inner-outer wall temperature difference ($-2\Delta T$).

In Figure 5, $(-M_{\rm t})/(-M_{\rm t})_{\Delta T}$ reaches its maximum value 0.956 at τ = 2.38 sec, which almost corresponds to the time when temperature difference between the outer wall and the average reaches its maximum. Note that the non-dimensional value $(-M_{\rm t})/(-M_{\rm t})_{\Delta T}$ is independent of initial wall-fluid temperature difference ($2\Delta T$) and material constants E, α, and ν.

Next, the transient SIF $K_{\rm cyl}$ of the circumferential crack (a/W = 0.5) in this long cylinder subjected to the $T(\tau, r)$ under discussion was calculated by Eq. (8) and compared with the SIF $K_{\rm Nied}$ analyzed by Nied and Erdogan [4] in Figure 6. In this comparison, these SIFs were normalized by the following SIF $K_{\rm tMax}$,

$$K_{\rm tMax}=F_{\rm tcyl}\cdot\left[\frac{(-M_{\rm t})_{\rm Max}}{Z}\sqrt{\pi a}\cdot F_M(\xi)\right] \quad (19)$$

which is the $K_{\rm tcyl}$ (Eq. (11)) for $(-M_{\rm t})_{\rm Max}$. $(-M_{\rm t})_{\rm Max}$ is defined as the maximum of ($-M_{\rm t}$) in the process obtained by the $T(\tau, r)$ and Eq. (4). So, $K_{\rm tMax}$ can be interpreted as the simplified evaluation of the maximum transient SIF by replacing the transient stress distribution σ_M (obtained by $T(\tau, r)$ and Eq. (2)) with a linear stress distribution which gives a moment equivalent to $M_{\rm t}$. Hereafter, we will call $K_{\rm tMax}$ as a *simplified evaluation by* $(-M_{\rm t})_{\rm Max}$. It is clear in this simplified evaluation that if $T(\tau, r)$ can be regarded as linear (i.e., σ_M can be regarded as linear distribution) $K_{\rm tMax}$ can be regarded as the desired $(K_{\rm cyl})_{\rm Max}$. Note that these SIFs normalized by this $K_{\rm tMax}$ are independent of E, α and ΔT.

From Figure 6, the difference between transient SIF K_{cyl} by Eq. (8) and SIF K_{Nied} by Nied and Erdogan turned out to be less than 4%. Furthermore, the maximum value of the transient SIF $(K_{cyl})_{Max}$ appeared at $\tau = 2.56$ sec, which is close to $\tau = 2.38$ sec when $(-M_t)$ reached its maximum. In addition, $(K_{cyl})_{Max}/K_{tMax} = 0.978$, which shows the validity of *simplified evaluation by* $(-M_t)_{Max}$ in this case.

3. 2 Effect of cylinder length on the maximum transient SIF K_{tMax} by simplified method

Before proceeding to the study on the effects of various structural parameters and heat transfer coefficient on the maximum transient SIF $(K_{cyl})_{Max}$, the effect of cylinder length on the maximum transient SIF K_{tMax} by simplified method will be discussed here. In Eq. (19), which gives the definition of K_{tMax}, it can be seen that the effect of the cylinder length is limited to the function $F_{t\,cyl}$. So we differentiated $F_{t\,cyl}$ by (βH) to study the effect of cylinder length H on the SIF K_{tMax}, as follows.

$$\frac{\partial F_{tcyl}}{\partial(\beta H)} = \frac{2\beta D\cdot\Delta\lambda\cdot\sinh\beta H\sin\beta H}{\left[(\cosh\beta H-\cos\beta H)+\beta D\cdot\Delta\lambda(\sinh\beta H+\sin\beta H)\right]^2} \tag{20}$$

From this equation, the fact that $F_{t\,cyl}$ shows a peak value at $(\beta H) = n\pi$ (n: integer) can be read. As there are many peaks, the maximum of these peaks will be determined.

First, $(F_{t\,cyl})_\infty$, which is a limit value of $F_{t\,cyl}$ for an infinitely long cylinder ($\beta H\to\infty$) was considered, as a reference value. Based on the knowledge of hyperbolic function,

$$\lim_{\beta H\to\infty}\sinh\beta H = \lim_{\beta H\to\infty}\cosh\beta H = \frac{\exp(\beta H)}{2} \tag{21}$$

and combining this with Eq. (12), $(F_{t\,cyl})_\infty$ is obtained as follows.

$$\left(F_{tcyl}\right)_\infty = \lim_{\beta H\to\infty}F_{tcyl} = \frac{1}{1+\beta D\cdot\Delta\lambda} \tag{22}$$

Though $(F_{t\,cyl})_\infty$ is affected by the crack length through $\Delta\lambda$, which is the increment of compliance due to the presence of crack, it is no more affected by the cylinder length. So, $F_{t\,cyl}$ normalized by this $(F_{t\,cyl})_\infty$ was represented in Figure 7 to intensify the characteristics of the peak values of $F_{t\,cyl}$ for cylinder length. In this figure, it can be seen that $F_{t\,cyl}$ (for respective crack lengths) increases for short H to reach the maximum at $H = \pi/\beta$, and gradually saturates to $(F_{t\,cyl})_\infty$ for long H.

3. 3 Effect of structural parameters and heat transfer coefficient on the maximum transient SIF

Next, effects of structural parameters and heat transfer coefficient on the maximum transient SIF $(K_{cyl})_{Max}$ were studied. The transient radial temperature distribution $T(\tau, r)$ of a finite-length hollow cylinder, which is insulated adiabatically outside and suddenly cooled inside from uniform temperature 0 K at time $\tau = 0$ sec by a coolant of temperature ($-2\Delta T$) = -100 K was calculated by Eq. (13) and (16). Heat transfer coefficient h was varied for 0.465, 1.16, 11.6 and ∞ kW/(m^2·K). The material constants of the cylinder are identical to those in the previous section. Cylinder configuration used in the study are as follows: R_m = 105 mm, W = 10 mm; H = 40, π/β = 79 and $5/\beta$ = 126 mm. For each H, crack length was taken at a = 1, 2, 3, 4, 5, 6 mm. The corresponding transient SIF K_{cyl} was calculated by Eq. (8). The maximum value $(K_{cyl})_{Max}$ of K_{cyl} was summarized in Figure 8 by using the Biot No. = hW/Λ and $K_{\Delta T}$ defined as follows:

$$K_{\Delta T} = \frac{E\alpha\Delta T}{1-\nu}\sqrt{\pi W} \quad \textbf{(23)}$$

From Figure 8, the *tendency* of the maximum transient SIF $(K_{cyl})_{Max}$ *to decrease for a long crack* can be read for all combination of cylinder length and heat transfer coefficient. In addition, $(K_{cyl})_{Max}$ was affected by the cylinder length and $(K_{cyl})_{Max}$ for a cylinder length of $H = \pi/\beta$ was larger than that for a long cylinder of $H = 5/\beta$. These are similar to the characteristics of the *SIF under linear temperature distribution* [3]. Furthermore, the change in $(K_{cyl})_{Max}$ due to crack length change becomes significant as the heat transfer coefficient becomes larger (the peak in Figure 8 turns out to be apparent).

Next, the validity of the *simplified evaluation by* $(-M_t)_{Max}$ (K_{tMax} in Eq. (19)) was checked for this case, for reference. The maximum transient SIF $(K_{cyl})_{Max}$ was normalized by K_{tMax} and summarized in Figure 9.

From Figure 9, the fact that the *simplified evaluation by* $(-M_t)$ gives the SIF on the safety side for a long crack ($a/W \geq 0.6$) can be read. On the other hand, this simplified evaluation gives the SIF on the dangerous side for up to approximately 50% for short cracks ($0.1 \leq a/W < 0.6$). The fact $(K_{cyl})_{Max}/K_{tMax}$ has a large discrepancy from unity for short cracks indicates that $(K_{cyl})_{Max}$ is strongly affected by the non-linearity of transient temperature distribution in the short crack region. Based on the knowledge that the stress distribution on the cracked surface contribute to the SIF, the reason non-linearity of transient temperature distribution significantly affects $(K_{cyl})_{Max}$ in the short crack region is considered to be the difference in the two linearized-temperature distributions (i.e., one on the crack surface and the other for the wall thickness).

Furthermore, the effect of cylinder length on $(K_{cyl})_{Max}$ was small in the short crack region. On the other hand, the effect of cylinder length on $(K_{cyl})_{Max}$ in the long crack region increased, though $(K_{cyl})_{Max}$ was not sensitive to the non-linearity of temperature distribution.

4. DISCUSSIONS

The SIF of an axisymmetric inner-surface circumferential crack in a finite-length, thin-walled and edge-restrained cylinder under non-linear radial temperature distribution was formulated by replacing the problem with two isothermal problems: (1) a pair of equal bending loads on cylinder edges and (2) non-linearly distributed stress on the crack surfaces. As the SIF for these isothermal problems were obtained based on the theory of cylindrical shell, beam and a method similar to the line spring method, accurate solution can be expected for thin (large R_m/W) and long (large H/W) cylinders. As a guideline for application, the expected difference between the SIF calculated by Eq. (8) and FEM is given as smaller than 5% for $R_m/W \geq 10.5$, $a/W \leq 0.6$, $H/W \geq 4$ [3], [5].

The temperature distribution treated in this paper is radial and not disturbed by the presence of the crack. Though this may sound to be unrealistic, it seems to be a good assumption in a practical sense. As the velocity of the fluid on the crack surface compared with that of main flow is small in our problem, and as the heat transfer coefficient is proportional to the power of the velocity, heat transfer on the crack surface is supposed to be negligibly small. So the heat flow in the axial direction is negligible and the temperature distribution near the crack will not be disturbed by the presence of the crack.

As the SIF of the crack under uniform temperature distribution treated is zero, maximum transient SIF is equal to the SIF range for a cycle. Thus, it can be said that the SIF range shows a *tendency to decrease for long crack*. If crack propagation rate fits Paris law when thermal stress for the problem we studied is repeated, crack arrest tendency can be justified.

5. CONCLUSION

In this paper, we formulated the transient SIF of the axisymmetric circumferential crack in a hollow cylinder, which is suddenly cooled inside from uniform temperature distribution. The cylinder was finite-length, thin-walled, adiabatically insulated outside and edges rotation-restrained. The crack was located at the midpoint of the cylinder length. By applying this method, the effects of various structural parameters (including cylinder length) and heat transfer conditions on the SIF can be easily considered. After formulation, the effects of structural parameters and heat transfer condition on the maximum transient SIF were illustrated numerically. The results showed that the maximum transient SIF for the problem decreased monotonously when the crack length became longer than a specific value for all combination of cylinder length and heat transfer coefficient. This corresponds to the tendency toward crack arrest. Furthermore, the SIF was strongly affected by the cylinder length, and the SIF for cylinder length of $H = \pi/\beta$ was larger than that for a long one. These characteristics of maximum transient SIF of the crack was similar to those for the SIF of the crack under linear radial temperature distribution.

So, for reference, the simplified method to evaluate the maximum transient SIF by the maximum transient equivalent thermal moment $(-M_t)_{Max}$ was tried and its validity was checked for R_m/W=10.5. This method can be considered as the linearization of the transient temperature distribution. As a result, the simplified method gave the SIF on the safety side for a long crack ($a/W \geq 0.6$).

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APPENDIX

(1) β which appears in the text of this paper is a parameter defined in replacing a problem of a cylinder with axisymmetric bending loads on its edges (Fig. A1 left) with the problem of a beam on an elastic foundation with bending loads on its ends (Fig. A1 right). β is defined as follows, by formally writing the flexural rigidity of the prismatic beam as $D = EW^3/12(1-\nu^2)$ [10],

$$\beta^4 = \frac{EW}{4R_m^2 D} = \frac{3(1-\nu^2)}{(R_m W)^2} \quad \textbf{(A 1)}$$

where R_m : mean radius, W : wall thickness, E : Young's modulus and ν : Poisson's ratio. Note that $1/\beta$ has a dimension of length.

(2) The weight function $w(x; a)$ used for evaluating K_{freec} is as follows [5]:

$$\begin{aligned} &w(x;a)\cdot[\sqrt{\pi}a^2W\sqrt{a-x}\cdot\psi_f\cdot F_M] \\ &=\sqrt{2}x\cdot F_M\cdot[Wx\cdot\psi_f+2a(a-x)\cdot\frac{\partial\psi_f}{\partial\xi}] \\ &+(a-x)\times\{2a\cdot\psi_f\cdot(\sqrt{2}x\cdot\frac{\partial F_M}{\partial\xi}+(a-x)\cdot\frac{\partial V}{\partial\xi})+V\cdot[W(2a+x)\cdot\psi_f+2a(a-x)\cdot\frac{\partial\psi_f}{\partial\xi}]\} \end{aligned} \quad \textbf{(A 2)}$$

where a: crack length (ξ=a/W), x: location on the crack surface (Figure 1), F_M and V: infinite length edge cracked beam's correction factor for finite width and dimensionless crack mouth opening under pure bending. ψ_f was a function deduced as follows:

$$\psi_f=\frac{\sinh\beta H+\sin\beta H}{\sinh\beta H+\sin\beta H+\beta D\cdot\Delta\lambda(\cosh\beta H+\cos\beta H-2)} \quad \textbf{(A 3)}$$

Note that the weight function was derived to include the effect of cylinder length H on the SIF. This effect is included by the introduction of ψ_f.

(3) The actual F_M, $\Delta\lambda$ and V used in the numerical calculations are given by the following equations (A 4) [11], (A 5) [12] and (A 6) [11], respectively.

$$F_M(\xi)=1.122-1.40\xi+7.33\xi^2-13.08\xi^3+14.0\xi^4 \quad \textbf{(A 4)}$$

$$\Delta\lambda(\xi)=\frac{\pi(1.1215)^2}{2E}\cdot\frac{\xi^2}{(1-\xi)^2(1+2\xi)^2}\times[1+\xi(1-\xi)(0.44+0.25\xi)]\left(\frac{6}{W}\right)^2 \quad \textbf{(A 5)}$$

$$V(\xi)=0.8-1.7\xi+2.4\xi^2+0.66/(1-\xi)^2 \quad \textbf{(A 6)}$$

LIST OF FIGURES

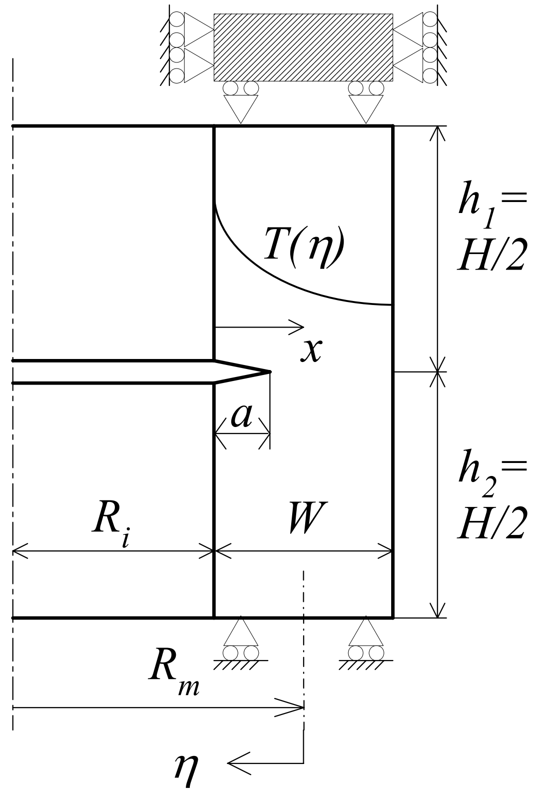

Figure 1 A cylinder with a circumferential crack under non-linear radial temperature distribution

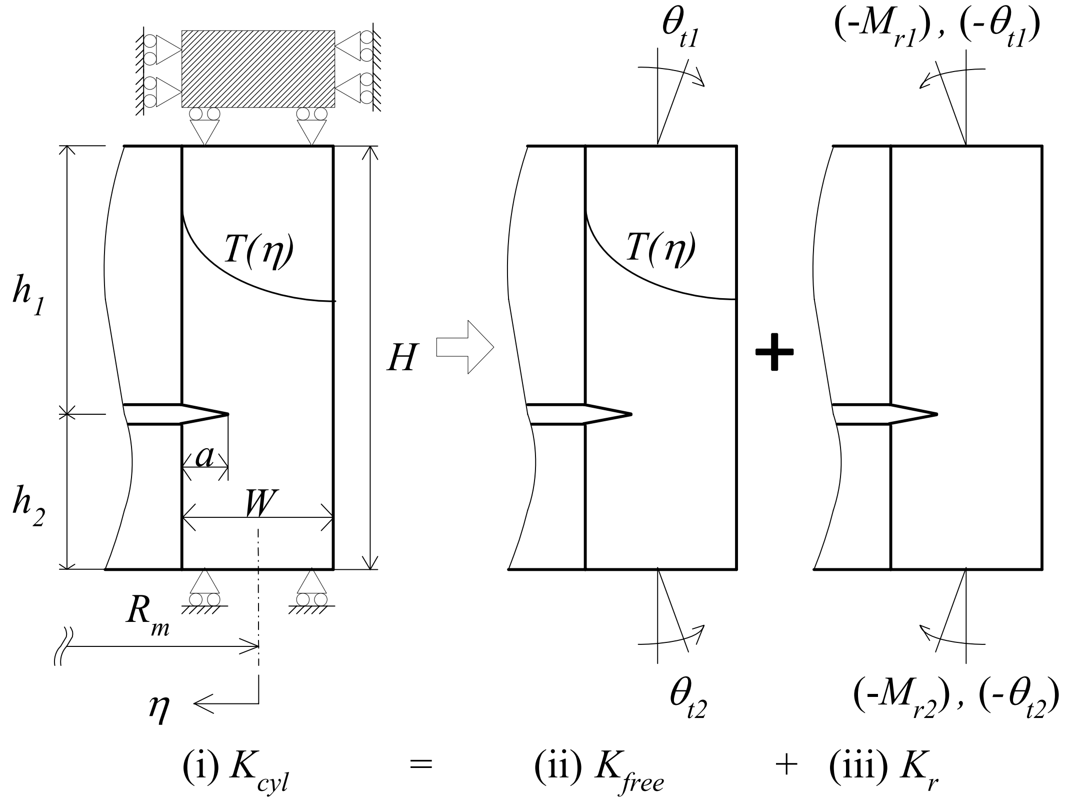

Figure 2 Principle of superposition

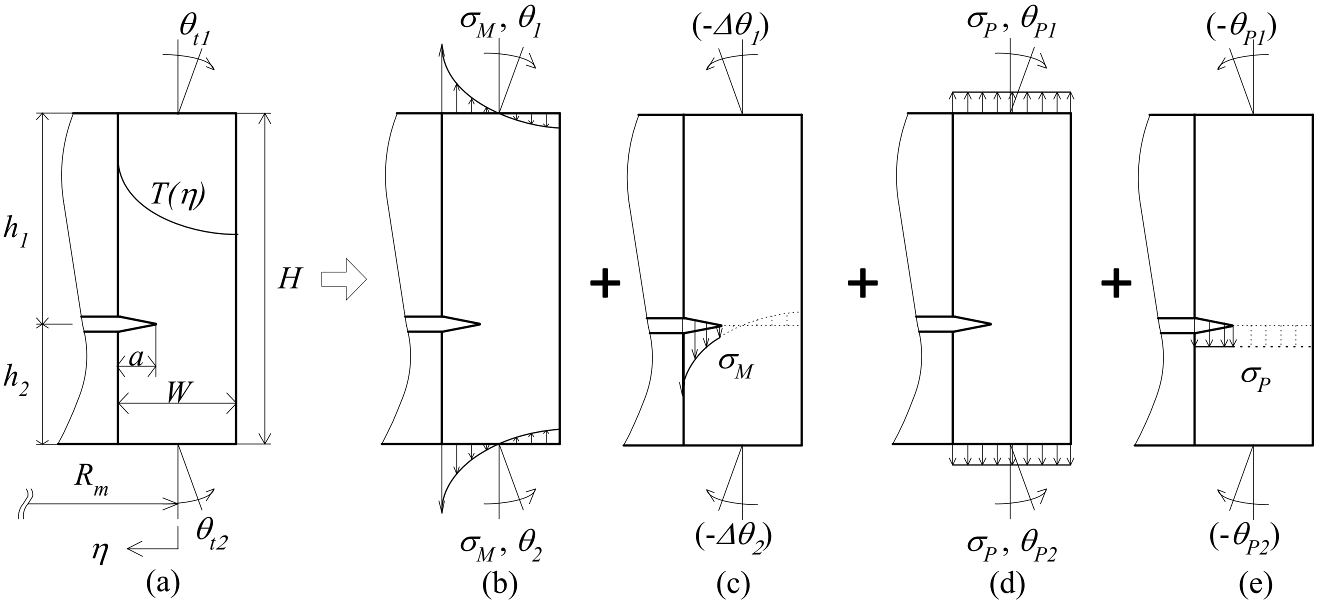

Figure 3 Duhamel's Analogy

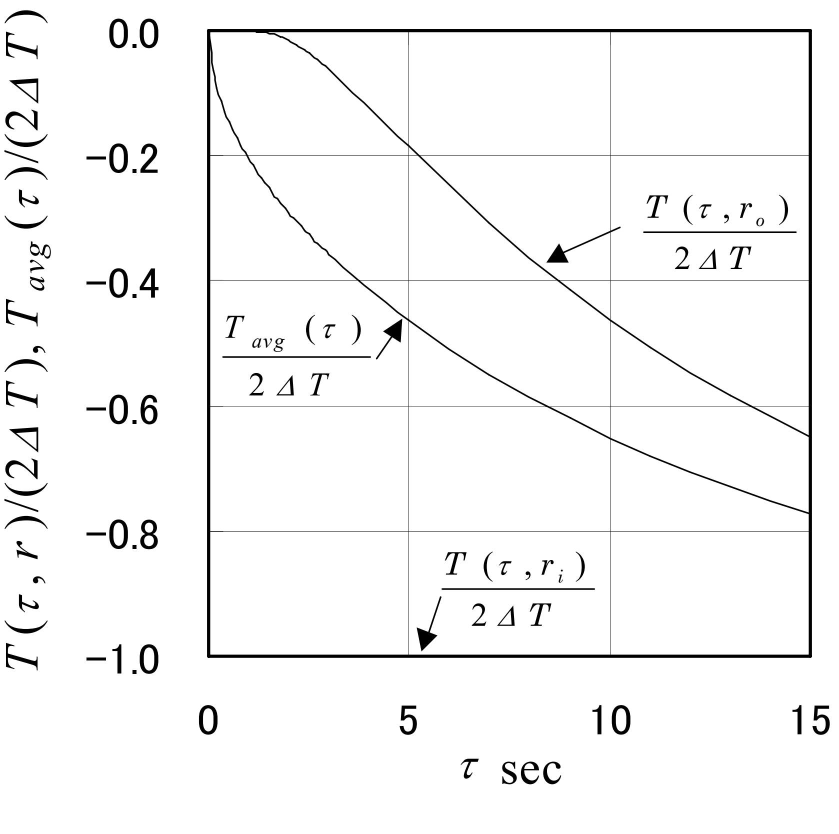

Figure 4 Transient temperature distribution (R_m/W = 9.5、W = 10 mm、h = ∞ W/(m^2·K))

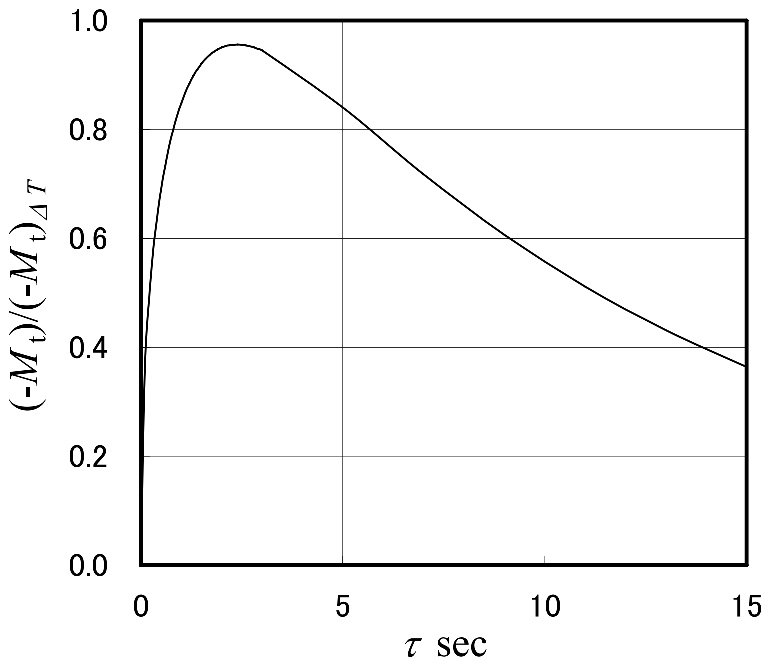

Figure 5　Transient equivalent thermal moment (-M_t) (R_m/W = 9.5、W = 10 mm、h = ∞ W/(m^2·K))

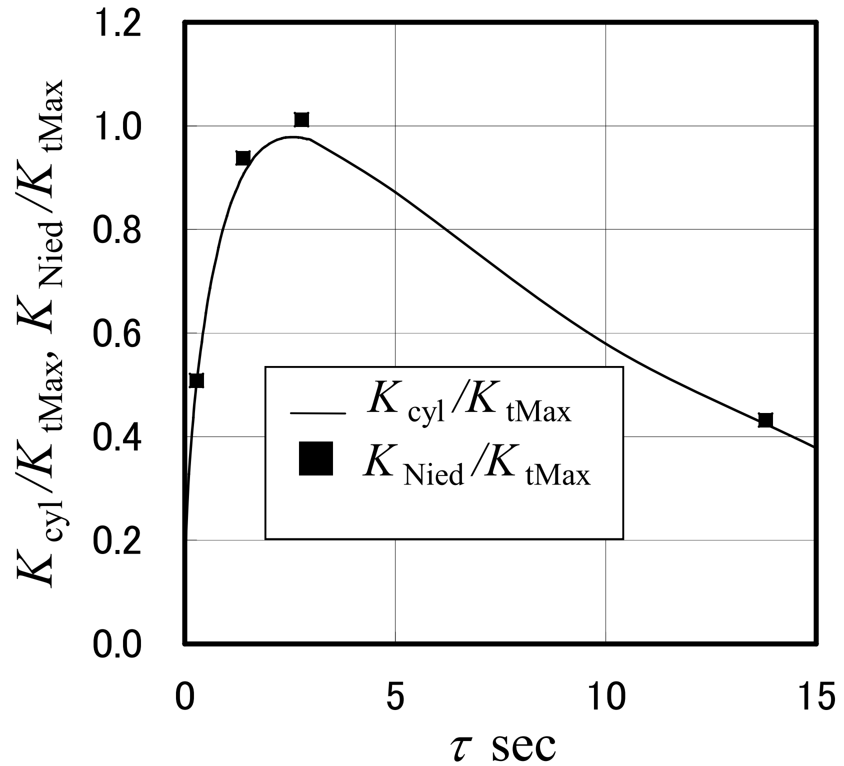

Figure 6　Transient stress intensity factor

(R_m/W = 9.5、H/W = 5/(βW) = 12.6、a/W = 0.5、W = 10 mm、h = ∞ W/(m^2·K))

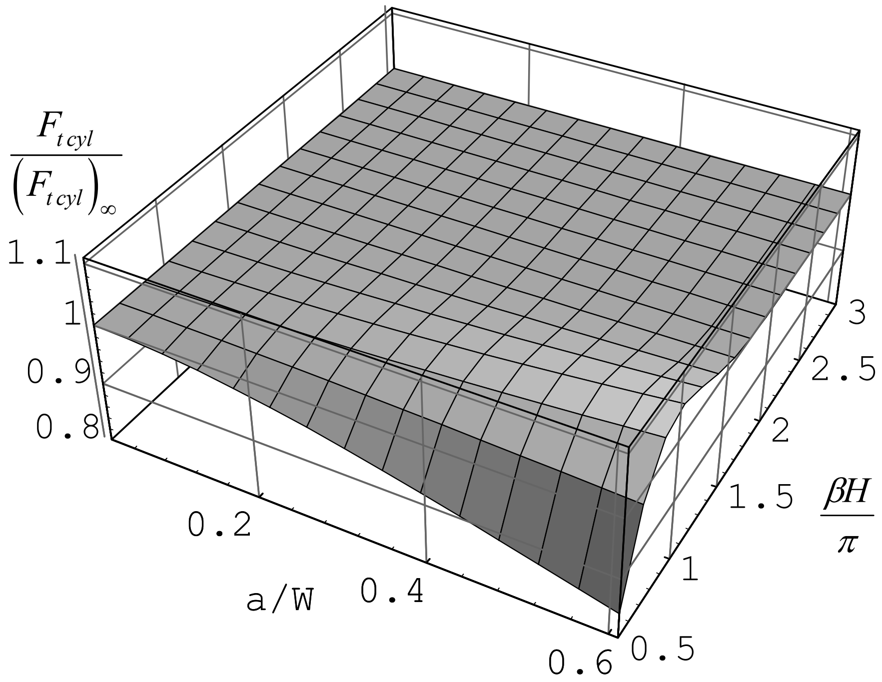

Figure 7 Effect of cylinder length on function $F_{t\,cyl}$ (R_m/W = 10.5、ν = 0.3)

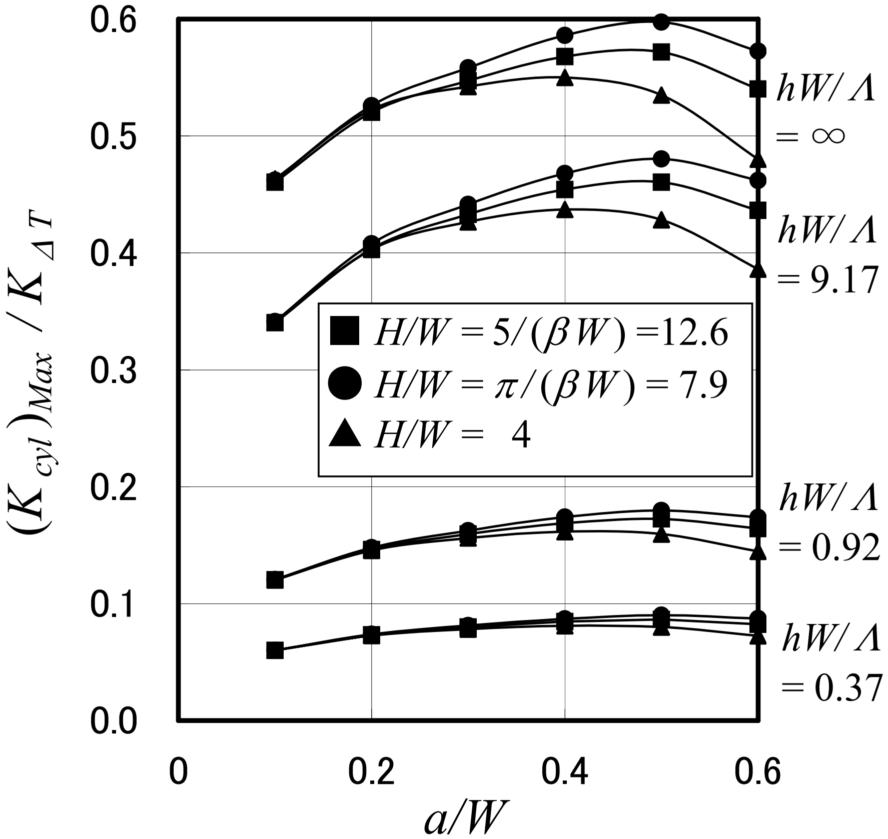

Figure 8 Maximum transient stress intensity factor $(K_{cyl})_{Max}$ (R_m/W = 10.5、W = 10 mm)

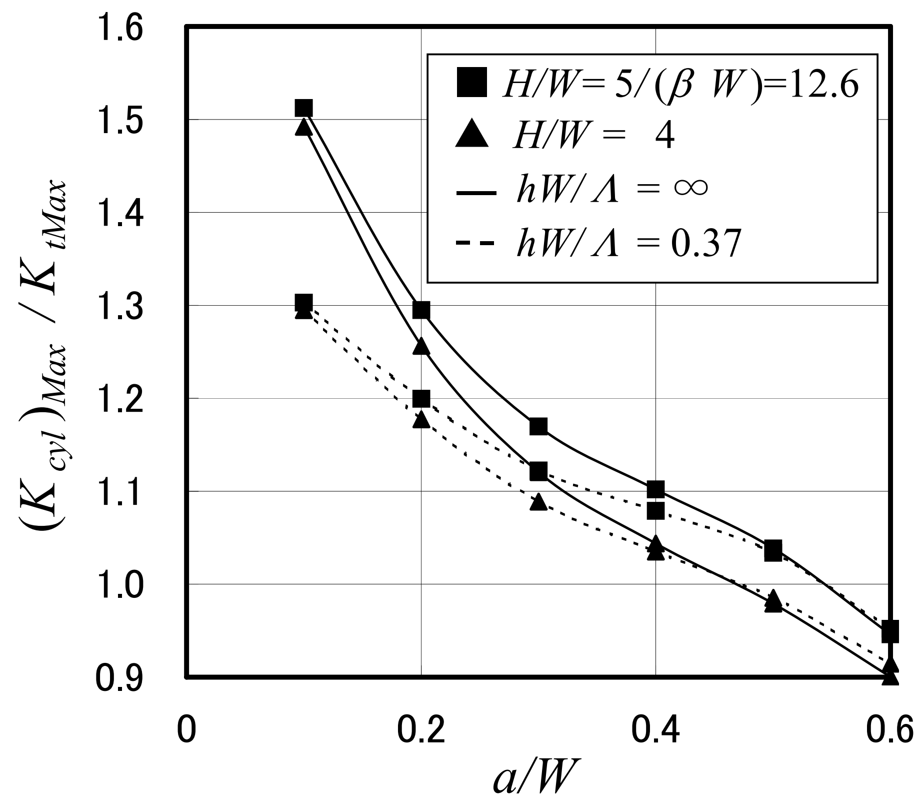

Figure 9 Comparison of $(K_{cyl})_{Max}$ with K_{tMax} (R_m/W = 10.5、 W = 10 mm)

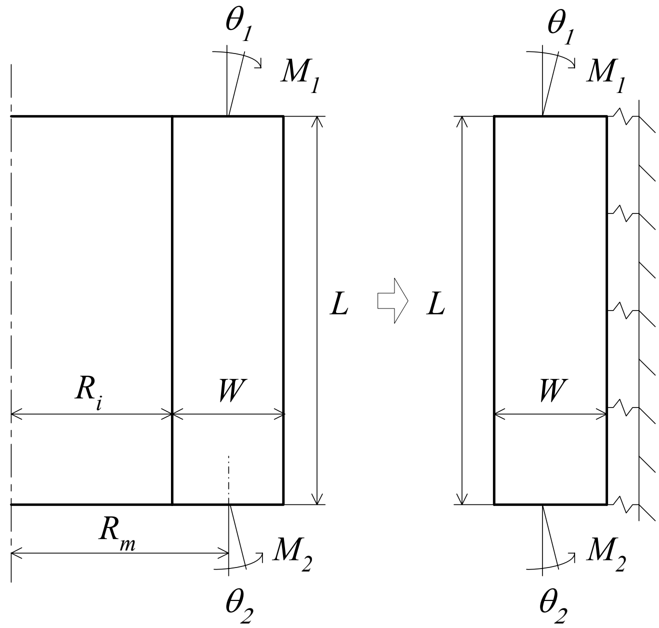

Fig. A1 Replacement of axisymmetric bending problem of a cylinder by a beam on an elastic foundation