Fixed-to-Variable Length Lossless Codes with Multiple Code Tables Considering Decoding Delay

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博 士 論 文 内 容 の 要 旨

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1　論文題目（英文の場合は，和訳を付記すること）

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（和訳：復号遅延を考慮した複数の符号表を用いる無歪み可変長符号）

2　要　　旨（和文 2,000 字程度又は英文 800 語程度にまとめること。）

Lossless source coding is a technology to represent given data in shorter bit lengths than the original representation without losing its contents. This enables efficient recording of data on storage devices and high-speed transmission of data over networks and plays an essential role in the advanced information society.

Huffman coding is a widely used lossless source coding methods in various applications such as image compression (JPEG) and video compression (H.264). The coding scheme of Huffman coding is described as the following system consisting of a source, an encoder, and a decoder: the source outputs a source sequence, a sequence of source symbols in the source alphabet, where each output symbol follows an independent and identical distribution; the encoder encodes each symbol of the source sequence to a binary codeword according to the code table obtained by Huffman's algorithm; then the decoder receives the codeword sequence, which is the concatenation of the codewords, and recovers the original source sequence from the codeword sequence.

The decoder is not given explicit information on the delimitation between the codewords in the codeword. However, the decoder can uniquely identify the delimitation by reading the codeword sequence from the beginning of it because of the following prefix-free property of the Huffman code table: no codeword is a prefix of any other codeword. The decoder can decode each codeword without any decoding delay as long as the encoder uses a prefix-free code table. For this reason, a prefix-free code is also called an instantaneous code. Huffman code is an

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instantaneous (prefix-free) code with the optimal average codeword length, a measure of compression performance, for a given source distribution.

However, it is known that one can achieve a better compression performance than Huffman coding by using a time-variant encoder with multiple code tables and allowing some decoding delay. AIFV (almost instantaneous fixed-to-variable length) codes developed by Yamamoto, Tsuchihashi, and Honda can attain a smaller average codeword length than Huffman codes by using a time-variant encoder with two code tables and allowing at most 2-bit decoding delay. Moreover, AIFV codes are generalized to AIFV-m codes, which can achieve a smaller average codeword length than AIFV codes for $m \geq 3$, allowing m code tables and at most m-bit decoding delay.

In this thesis, we discuss a more general class of source codes with multiple code tables considering decoding delay than AIFV-m codes and show their properties. We first formalize source codes with a finite number of code tables as code-tuples, and then we introduce two equivalent definitions of k-bit delay decodable code-tuples, which allow at most k-bit decoding delay for $k \geq 0$. Then we prove three theorems related to k-bit delay optimal code-tuples, which are defined as code-tuples with the optimal average codeword length for a given source distribution among all the k-bit delay decodable code-tuples. These theorems describe properties of k-bit delay decodable code-tuples by the set of the possible first k bits of the codeword sequence in the case of starting from each code table.

The first theorem claims that there is no need for more than one code table such that the sets of the possible first k bits of the codeword sequence are equal. This implies that it is not the case that one can achieve an arbitrarily small average codeword length by using arbitrarily many code tables, and it is sufficient for us to consider at most finitely many code tables. In particular, this guarantees that a k-bit delay optimal code-tuple does indeed exist. Also, the first theorem gives a concrete upper bound of the required number of code tables for a k-bit delay optimal code-tuple.

The second theorem gives the following necessary condition for a k-bit delay decodable code-tuple to be optimal: if the first k bits of a given binary sequence is a prefix of some codeword sequence, then the entire given binary sequence is also a prefix of some codeword sequence. This result is a generalization of the property of Huffman codes that each internal node in the code tree has two child nodes.

The third theorem shows that it is sufficient to consider only the code-tuples such

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that both 0 and 1 are possible as the first bit of codeword no matter which code table we start the encoding process from.

These three theorems enable us to limit the scope of codes to be considered when discussing k-bit delay optimal codes in theoretical analysis and practical code construction.

As applications of the three theorems, for k = 1, 2, we give a class of k-bit delay decodable code-tuples which include a k-bit delay optimal code-tuple for a given source distribution. More specifically, we first prove that the Huffman code achieves the optimal average codeword length in the class of 1-bit delay decodable code-tuples. Namely, the class of instantaneous codes with a single code table can achieve the optimal average codeword length in the class of 1-bit delay decodable code-tuples. Then we also prove that the class of AIFV codes can achieve the optimal average codeword length in the class of 2-bit delay decodable code-tuples. In particular, this result implies that it is sufficient to consider at most two code tables to find a 2-bit delay optimal code-tuple.