A Fuzzy Classifier System Using Hyper-Cone Membership Functions and Its Application to Inverted Pendulum Control

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Abstract— **This paper proposes a fuzzy classifier system (FCS) using fuzzy rules given by hyper-cone membership functions. The hyper-cone membership function is expressed by a kind of radial basis function, and its fuzzy rules can be flexibly located in input and output spaces. Therefore, The FCS can generate excellent rules which have the best location and shape of membership functions. We apply the FCS to a fuzzy rule generation for the inverted pendulum control. Also, we introduce the simplified reward acquisition method for evaluation of inverted pendulum performance.**

Keywords— **Fuzzy Classifier System, Fuzzy Rule Generation, Genetic Algorithm, Inverted Pendulum.**

I. Introduction

Fuzzy systems using fuzzy reasonings have been applied in various fields such as fuzzy control. However, there are tuning problems in membership functions and reasoning rules. Also, it is difficult to obtain fine fuzzy rules showing best performance for the system. Therefore, the study for automation of this processes have led to many researches with various tools for system development. For example, there are neural networks[1]–[3], genetic algorithms (GAs)[4]–[12], clusterings[13],[14] and so on.

Fuzzy rules generation methods using GAs can include the framework of genetics-based machine learning (GBML). In the GBML, there are two frameworks of the Pits approach and the Michigan approach. Almost fuzzy rule generation methods by GA are classified into Pits approach[4]–[10]. In these methods, however, much genetic information is necessary, and it is difficult to apply to large-scale system because many fuzzy systems are necessary in GA coding.

One the other, fuzzy classifier systems (FCSs) applying the Michigan approach are proposed. Valenzuela-Rendon proposed FCSs and applied it to an approximation of a single-input single-output equation[11]. Furuhashi et al. expanded multiple FCSs and applied it to obtaining fuzzy rules of a ship control[12]. In these methods, GA was done in one fuzzy system, and effective fuzzy rules were found, because one rule is made to be an individual. However, these methods are fixed membership functions. Therefore, these are methods which choose necessary fuzzy rules among the large number of rules without tuning of the membership functions.

A purpose of our study is the development of the automatic generation technique of fuzzy rules by FCSs with the decision of the rule number, location and rule shape. We presented automatic generation methods of fuzzy rules using hyper-cone membership functions by the Pits approach style GAs[8],[9]. The hyper-cone membership function is expressed by a kind of radial basis function, and its fuzzy rule can be flexibly located in input and output spaces. In this paper, we propose an automatic generation method of fuzzy rules using hyper-cone membership functions by FCS. We apply this method to obtaining fuzzy control rules of the inverted pendulum system. Also, we introduce the simplified reward acquisition method for evaluation of performance of the inverted pendulum.

II. Fuzzy Rules Using Hyper-Cone Membership Functions

A. Hyper-Cone Membership Functions

In this method, we give fuzzy rule R^i as below:

$$R^i : \text{if } \boldsymbol{x} \text{ is } A_i \text{ then } \boldsymbol{y} \text{ is } B_i,\ i = 1, 2, \cdots, n \quad (1)$$

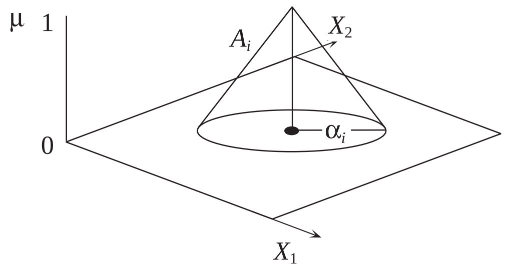

Fig. 1. Shape of hyper-cone membership function ($l = 2$)

where i is rule number, n is the number of rules, $\boldsymbol{x}$ and $\boldsymbol{y}$ are the input and output vectors, respectively, and A_i and B_i are fuzzy subsets. In Eq.(1), the input vector $\boldsymbol{x}$ and the output vector $\boldsymbol{y}$ are given by

$$\boldsymbol{x} = \left[\begin{array}{cccc} x_1 & x_2 & \cdots & x_l \end{array}\right]^{\mathrm{T}}, \boldsymbol{y} = \left[\begin{array}{cccc} y_1 & y_2 & \cdots & y_m \end{array}\right]^{\mathrm{T}} \quad (2)$$

where l and m are the number of input and output variables, respectively.

A_i and B_i of the rule R^i are expressed by hyper-spherical fuzzy subsets directly corresponding to a subspace in input and output spaces. Therefore, fuzzy subsets A_i and B_i are defined by a hyper-cone membership function described below. In this fuzzy system, since there are n fuzzy rules, n hyper cone membership functions are located in input and output each space.

The hyper-cone membership function $\mu_{A_i}(\boldsymbol{x})$ which expresses A_i is defined by Eqs.(3) and (4) in input space.

$$\mu_{A_i} : A_i \to [0, 1] \quad (3)$$

$$\mu_{A_i}(\boldsymbol{x}) = \left(1 - \frac{\|\boldsymbol{x} - \boldsymbol{a}_i\|}{\alpha_i}\right) \vee 0 \quad (4)$$

where $\boldsymbol{a}_i$ and α_i are the center vector and the radius of the fuzzy subsets A_i. The membership function μ_{A_i} has a grade 1.0 at the center $\boldsymbol{a}_i \in \boldsymbol{R}^l$ of the fuzzy subset A_i whose radius is α_i. The membership value decreases in proportion to the distance from the center $\boldsymbol{a}_i$. At the circumference of this sphere, a grade has 0.0. Fig.1 shows the hyper-cone membership function in case of $l = 2$. Hyper-cone membership function μ_{B_i} is defined in m dimensional output space by the same way as the input membership function.

B. Reasoning Method

We find the reasoning result μ_{B^*} from the input and output membership functions, defuzzy it, and calculate the real output value $\boldsymbol{y}^*$.

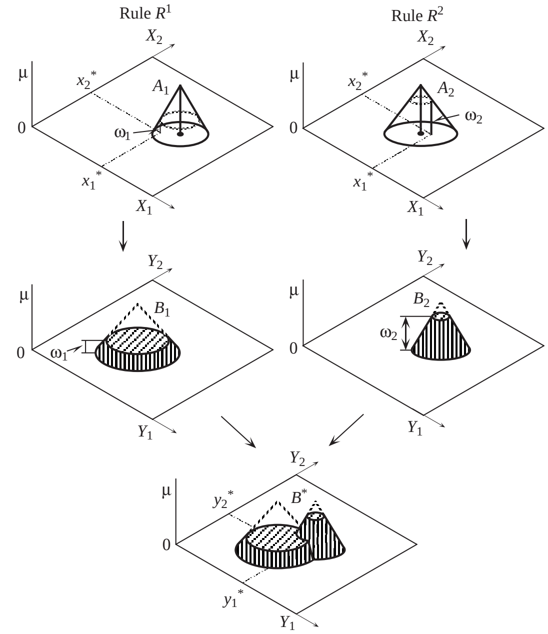

Fig. 2. An example of the reasoning method

In the first step, the truth value ω_i of antecedent part in a rule R^i for input vector $\boldsymbol{x}^*$ is calculated by Eq.(4). In other words, the membership value is the truth value ω_i. In the second step, we use the truth value ω_i to define a membership function $\mu_{B_i{}^*}(\boldsymbol{y})$ as shown in Eq.(5).

$$\mu_{B_i{}^*}(\boldsymbol{y}) = \omega_i \wedge \mu_{B_i}(\boldsymbol{y}) \quad (5)$$

The shape of membership function $\mu_{B_i{}^*}(\boldsymbol{y})$ is the truncated $\mu_{B_i}(\boldsymbol{y})$ at ω_i. We find the composite reasoning result $\mu_{B^*}(\boldsymbol{y})$ for each rule in Eq.(6).

$$\mu_{B^*}(\boldsymbol{y}) = \bigvee_{i=1}^{n} \mu_{B_i^*}(\boldsymbol{y}) \quad (6)$$

In the final step, output $\boldsymbol{y}^*$ is given by the center of gravity of the membership function $\mu_{B^*}(\boldsymbol{y})$.

$$\boldsymbol{y}^* = \frac{\int_{D_y} \mu_{B^*}(\boldsymbol{y}) \boldsymbol{y} d\boldsymbol{y}}{\int_{D_y} \mu_{B^*}(\boldsymbol{y}) d\boldsymbol{y}} \quad (7)$$

Fig.2 shows an example of the reasoning in the case of $l = 2,\ m = 2,\ n = 2$.

In our method, all rules can not always cover input space. Therefore, there often exists spaces whose membership grade for input $\boldsymbol{x}^*$ is zero. If an input vector $\boldsymbol{x}^*$ is determined in such a space, a rule R^ϕ to be fired is defined by

$$d_i = \|\boldsymbol{x}^* - \boldsymbol{a}_i\| - \alpha_i \quad (8)$$

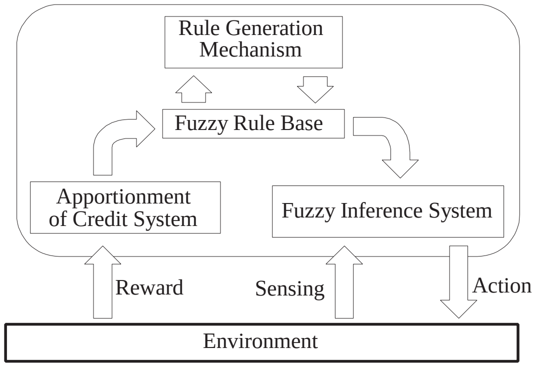

Fig. 3. Fuzzy Classifier System

$$R^{\phi} = \left\{ R^i \left| \min \left\{ d_i \right\} \right. \right\} \tag{9}$$

Eqs.(8) and (9) mean that only the rule having the membership function closest to input vector $\boldsymbol{x}^*$ is fired with $\omega_\phi = 0.0$. In case of R^ϕ, $\boldsymbol{y}^*$ is given the center coordinate of fuzzy set B_ϕ.

III. Fuzzy Classifier System

A. Overview of Fuzzy Classifier System

FCS consists of four blocks of "Fuzzy Rule Base", "Fuzzy Inference System", "Apportionment of Credit System" and "Rule Generation Mechanism" as shown in Fig.3. Fundamental operations of FCS are as follows.

[Fuzzy Rule Base]

In the Fuzzy Rule Base, suppose there existed n fuzzy rules. Using these rules, fuzzy reasoning is carried out in Fuzzy Inference System.

[Fuzzy Inference System]

In the Fuzzy Inference System, senses (inputs) are received from an environment, and fuzzy rules which suit these senses are chosen from the Fuzzy Rule Base. Then, fuzzy reasoning is carried out from those rules, and actions (outputs) to the environment are decided.

[Apportionment of Credit System]

In the Apportionment of Credit System, rewards are provided from the environment to the FCSs for actions which were decided in the Fuzzy Inference System. Also, rewards are distributed to each rule as a credit. The reward is an evaluation from the environment for the action. In other words, it is an evaluation for the fuzzy system. The credit is an evaluation of each rule, and higher the rule contributes to obtain the reward, higher the evaluation of the rule increases.

[Rule Generation Mechanism]

New rules are generated based on credit cf_i by GA. A coding of one rule in this GA is done as one individual. Therefore, each credit is given a fitness of each individual. In this GA method, population is composed of n individuals because there are n fuzzy rules. The population in next generation is generated base on this population by selection, crossover and mutation.

B. Fuzzy Classifier Systems Using Hyper-Cone Membership Functions

Conventional FCSs beforehand define membership functions of each variable, and search the necessary rules by GAs[11],[12]. Therefore, these methods may dependent on a membership functions shape and the number. In this method, the Fuzzy Rule Base consisted of fuzzy rules using hyper-cone membership functions is introduced in the FCSs. In this fuzzy system, it is possible to appropriately locate fuzzy rules, because shape and location of membership functions can be handled as parameters of each fuzzy rule.

Four blocks of the proposal FCS method is considered as following.

[Fuzzy Rule Base]

There exist n fuzzy rules using hyper-cone membership functions.

[Fuzzy Inference System]

Using these rules, fuzzy reasoning with the method described in the chapter II is carried out.

[Apportionment of Credit System]

The approach of this block is equal to the normal method. Rewards are provided from the environment, and each rule distributes there as a credit. In this presented method, the credit cf_i of each rule is provided following method. When there are J actions in one trial, a reward $re_j (j = 1, 2, \ldots, J)$ is given in each action. The credit $cf_i (i = 1, 2, \ldots, n)$ is an evaluation of each rule, and higher the rule contributes to obtain the reward, higher the evaluation of the rule increases. Therefore, the credit $cf_i (i = 1, 2, \ldots, n)$ is given from the reward in proportion to the truth value of the rule. The credit cf_i is given as follows:

$$cf_i = \sum_{j=1}^{J} \frac{\mu_{ij}}{g_j} \times re_j \tag{10}$$

$$g_j = \sum_{i=1}^{N} \mu_{ij} \tag{11}$$

where μ_{ij} is the truth value of fuzzy rule R^i in reasoning action (output) j.

[Rule Generation Mechanism]

Next generation fuzzy rules expressed hyper-cone membership functions are generated by GA. Genetic information of fuzzy rule R^i is location and

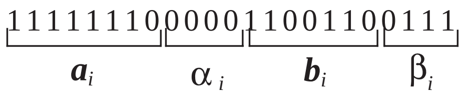

Fig. 4. Example of chromosome

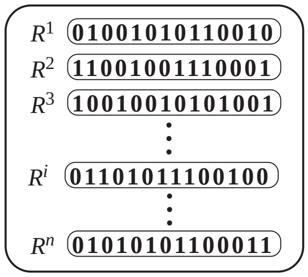

Fig. 5. Composition of population

shape of input and output membership function. Therefore, parameters of hyper-cone membership functions composed one fuzzy rule are searched by GA.

Genetic parameters of fuzzy rule R^i are as follow

- Center coordinate $\boldsymbol{a}_i$ of fuzzy subset A_i,
- Radius α_i of fuzzy subset A_i,
- Center coordinate $\boldsymbol{b}_i$ of fuzzy subset B_i, and
- Radius β_i of fuzzy subset B_i.

These parameters are coded as one chromosome like Fig.4. In other words, one fuzzy rule is expressed by one chromosome, and one chromosome is one individual. Since there are n fuzzy rules, population is composed of n chromosomes (see Fig.5).

This method is used genetic operations based on the simple GAs. New fuzzy systems are generated by selection, crossover and mutation operations (see Fig.6). The fitness of each rule is the credit value provided to each rule. The procedure of genetic operations is as follows:

[Step1] An initial population is randomly produced. Also, the fitness (the credit cf_i) of each individual (rule) is calculated.

[Step2] We produce the population of the next generation the by following operation.

[Step2-1] In Selection, two individuals (rules) are selected by the roulette wheel model to cross each other.

[Step2-2] In Crossover, two individuals cross each other. In this method, one point crossover method is used.

[Step2-3] In Mutation, the mutation rate is constant at each gene.

[Step2-4] We add two individuals (fuzzy rules) generated by the above procedure to the population of the next generation. If the number of individuals in the next population is n, then we go to Step3. If the number of individuals in the next population is under n, then we go back to Step2-1.

[Step3] Carrying out the new fuzzy system (new population), rewards are provided from the environment, and the credit cf_i of each rule is calculated. In other word, fitness values are provided.

[Step4] If a prespecified stopping condition is not satisfied, return to Step2. If this condition is satisfied, this process ends.

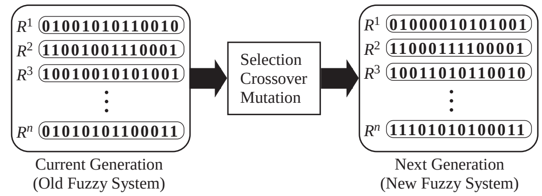

Fig. 6. Genetic operations

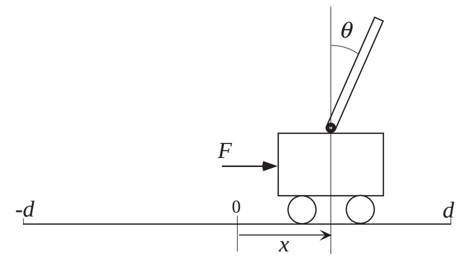

Fig. 7. Model of Inverted Pendulum

IV. Application of FCS to Inverted Pendulum Control

A. Simulation Model of Inverted Pendulum

We generated fuzzy rule for the inverted pendulum system by the presented method. The inverted pendulum is the classical non-linear control system. Fig.7 is showed its simulation model. In this figure, a cart is put on a rail from the center to both sides with d [m]. The friction between the rail and the cart and the friction of the drive system jointed pole on the cart are disregarded. In this system, the objective is to control the translational forces in order to position the cart at the center of finite width rail while balancing the pole on the cart simultaneously.

The state variables of the inverted pendulum are following

θ: The angle of pole for verticality [deg],
$\dot{\theta}$: The angular velocity of pole [deg/s],
x: The distance of the cart from center [m],
$\dot{x}$: The velocity of cart [m/s].

The dynamic equation for the inverted pendulum is expressed as follow:

$$\ddot{\theta} = \frac{(m_c + m_p) g \sin\theta - (F + m_p l \dot{\theta}^2 \sin\theta)\cos\theta}{\left\{\frac{4}{3}(m_c + m_p) - m_p \cos^2\theta\right\} l} \quad (12)$$

$$\ddot{x} = \frac{\frac{4}{3}(F + m_p l \dot{\theta}^2 \sin\theta) - m_p g \sin\theta\cos\theta}{\frac{4}{3}(m_c + m_p) - m_p \cos^2\theta} \quad (13)$$

where, $\ddot{\theta}$ is the angular acceleration of the pole [deg/s^2], $\ddot{x}$ is the acceleration of the cart [m/s^2], and F is the force [N] added to the cart. Also, in these equations, m_c is the weight of cart [kg], m_p is the weight of the pole [kg], l is the length of the pole [m], and g is the gravitational constant [9.8m/s^2].

Each state variable is calculated with the minute sampling period τ by the Runge-Kutta method. Therefore, Each variable of the sampling time $(k+1)\tau$ is expressed by Eq.(14).

$$\begin{cases} \theta(k+1) = \theta(k) + \tau\dot{\theta}(k) \\ \dot{\theta}(k+1) = \dot{\theta}(k) + \tau\ddot{\theta}(k) \\ x(k+1) = x(k) + \tau\dot{x}(k) \\ \dot{x}(k+1) = \dot{x}(k) + \tau\ddot{x}(k) \end{cases} \quad (14)$$

In this computer simulation, the sampling period τ is 0.02[s].

In generated fuzzy systems, inputs are θ, $\dot{\theta}$, x and $\dot{x}$, and output is F. Therefore, this application is four inputs and one output problem.

B. Reward Acquisition Method

The reward for the FCSs is given by evaluating the performance of the inverted pendulum simulation. In this case, if states of the inverted pendulum satisfy one of conditions of Eq.(15), the control is failed (the pole fell down, or the cart fell down from the rail), and the simulation is terminated.

$$\begin{cases} |\theta| > \theta_{max} \\ |x| > d \end{cases} \quad (15)$$

Simulations from multiple initial positions are evaluated in order to obtain controllable fuzzy systems from all positions. In this method, complicate evaluation functions are not used for system evaluation, but a simple reward acquisition method is introduced as follows:

Rewards are separately provided for the angle of the pole and the position of the cart every each time step. Therefore, the reward re_{j1} of the pole and the reward re_{j2} of the cart of the time step s are calculated as follows:

$$\Delta\theta = |\theta_{j-1}| - |\theta_j| \quad (16)$$

$$re_{j1} = \begin{cases} 1, & \Delta\theta \geq 0 \\ 0, & \Delta\theta < 0 \end{cases} \quad (17)$$

$$re_{j2} = \begin{cases} 1, & |x_j| \leq td \\ 0, & |x_j| > td \end{cases} \quad (18)$$

where td is parameter of evaluating the cart position. The re_{j1} is provided reward 1 if the angle of the pole is smaller than the time step before one. In other words, if the angle of the pole has been improved, it is a good action, and reward is given. Also, The re_{j2} gets reward 1 if the cart is to the position within td from the center.

The reward re_j of the time step j is caluculated by adding re_{j1} and re_{j2}.

$$re_j = w_1 re_{j1} + w_2 re_{j2} \quad (19)$$

where w_1 and w_2 are weights, respectively. The credit cf_i of each rule is calculated by Eq.(10).

This method is a comparatively simple method, because whether it has improved the state in the last time step or whether it keeps the state become criterion.

C. Simulation Result

Parameters of the inverted pendulum are set as follow;

$$\begin{aligned} l &= 0.5 \text{ m} \\ m_g &= 1.0 \text{ kg} \\ m_p &= 0.1 \text{ kg} \\ d &= 2.4 \text{ m} \\ \theta_{max} &= 12.0 \text{ deg} \end{aligned}$$

A chromosome representing each rule is composed of 34 bits genes and assigned as follows: each element (x, $\dot{x}$, θ and $\dot{\theta}$) of the center coordinate $\boldsymbol{a}_i$ of fuzzy subset A_i is 5 bits, the radius α_i of fuzzy subset A_i is 4 bits, the center coordinate $\boldsymbol{b}_i$ (F) of fuzzy subset B_i is 6 bits, and the radius β_i of fuzzy subset B_i is 4 bits.

We set the number of rules (the population size) $n = 15$. The crossover rate was 25%, the mutation rate was 3.0%, and the number of generation was 10000. The credit cf_i was given by evaluations of four initial positions ((θ_0, x_0) = (-10.0, -1.0), (-10.0, 1.0), (10.0, -1.0) and (10.0, 1.0)). Also, td was 0.8, and weights w_1 and w_2 were set for 1.0 and 0.5, respectively.

We tried ten times for different initial populations, and obtained fuzzy rule sets. Figs.8-10 show simulation results of fuzzy rule sets obtained by the FCS. In these figures, Fig.8 and Fig.9 are results of initial position using evaluating, and Fig.10 is result of another position. From these figures, our proposed method could control the inverted pendulum in such a way that the pole would not fall, and the angle of the pole has been converged on 0. Also, the cart skillfully approaches to the center of

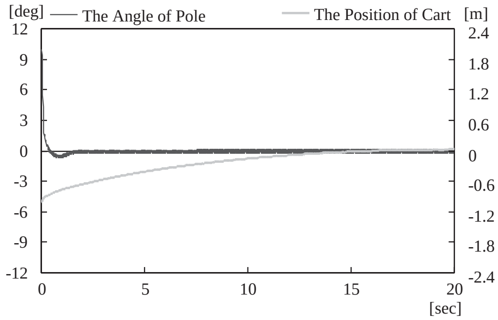

Fig. 8. Simulation result (θ_0, x_0) = (10.0, -1.0)

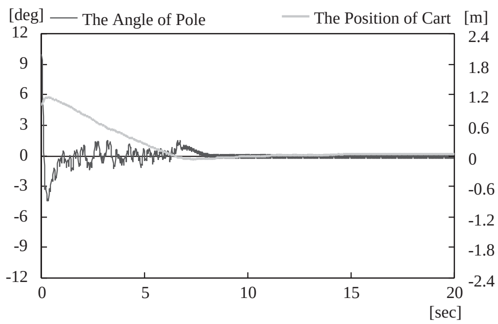

Fig. 9. Simulation result (θ_0, x_0) = (10.0, 1.0)

the rail. Moreover, expect from the edges of the rail, it was controllable from most position. This means that 15 rules using hyper-cone membership functions were placed for the appropriate position in input and output spaces by FCS, and obtained fuzzy systems are versatile systems. Therefore, the FCS using fuzzy rules expressed hyper-cone membership functions is effective in the design of fuzzy systems. Also, even if the reward acquisition system is simplified, useful fuzzy systems are obtained.

V. Conclusions

In this paper, we presented an automatic generation technique for fuzzy rules using hyper-cone membership functions by FCS, and applied this method to the inverted pendulum problem. Also, a simple reward acquisition method was introduced in this method. The simulation results using obtained fuzzy systems by FCS showed skillful performance.

There remains problems to be solved in future. In obtained 15 rules, overlapping rules and resembled rules were included. It is necessary to improve GA methods so that it may not become same rules. Also, methods which can decide the rule number are necessary. We must apply the proposed method to other problems and confirm the usefulness of this method.

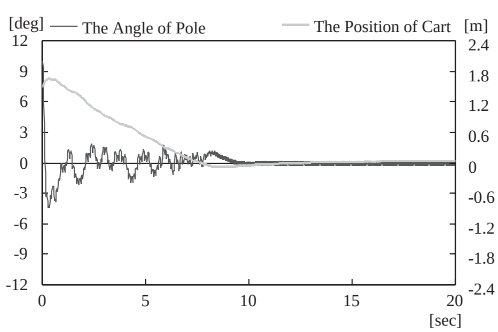

Fig. 10. Simulation result (θ_0, x_0) = (10.0, 1.5)

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