Three-dimensional T-stresses for three-point-bend specimens with large thickness variation

著者	Lu Kai, Meshii Toshiyuki
journal or publication title	Engineering Fracture Mechanics
volume	116
page range	197-203
year	2014
URL	http://hdl.handle.net/10098/8209

Three-dimensional T -stresses for three-point-bend specimens with large thickness variation

Kai Lu^{1*}, Toshiyuki Meshii²

¹ Graduate School of Engineering, University of Fukui, 3-9-1 Bunkyo, Fukui, Fukui, Japan

² Faculty of Engineering, University of Fukui, 3-9-1 Bunkyo, Fukui, Fukui, Japan

* Corresponding author: kai_lu@u-fukui.ac.jp FAX : +81-776-27-9764

Abstract

Three-point-bend (3PB) test specimens are useful for the systematic investigation of the influence of statistical and constraint loss size effects on the cleavage fracture toughness of a material in the ductile-to-brittle transition temperature range. Because the in- and out-of-plane elastic T -stresses (T_{11} and T_{33}) are a measure of the crack-tip constraint and even the in-plane T_{11} exhibits three-dimensional (3D) effects, the 3D T -stresses solutions were obtained by running finite element analyses (FEA) for 3PB specimens with wide ranges of the crack depth-to-width ratio ($a/W = 0.2$ to 0.8) and the specimen thickness-to-width ratio ($B/W = 0.1$ to 40). The results show that the 3D T_{11} at the specimen mid-plane tended to deviate from the 2D T_{11} as B/W increased, with the deviation saturating for $B/W \geq 2$. The mid-plane T_{33} increased with B/W and was close to the plane strain value $\bar{\sigma}_{11}$ for $B/W \geq 2$.

Keywords: Elastic T -stress, Three-point-bend specimen, Finite element analysis, Fracture toughness, Constraint effect

Nomenclature

B	Specimen thickness
E	Young's modulus
F	Unit magnitude (see Eq. (2))
I	Interaction integral
K_I	Local mode I stress intensity factor (SIF)
K_0	2D SIF for elastic analysis
R_s	Crack tube radius
S	Support span for 3PB specimen
T_{11}, T_{33}	T -stresses

W	Specimen width
a	Crack length
r, θ	In-plane polar coordinates
x_j	Crack-tip local coordinates ($j = 1, 2, 3$)
Δ	Singular element size
β_1, β_3	Normalized T -stresses
ε_3	Out-of-plane strain
ν	Poisson's ratio
σ	Stress components ($i, j = 1, 2, 3$)

1. Introduction

Three-point-bend (3PB) test specimens are useful for the systematic investigation of the statistical and constraint loss size effects on the cleavage fracture toughness of a material in the ductile-to-brittle transition temperature range [1, 2]. Because the in-plane and out-of-plane T -stresses (T_{11} and T_{33}) are a measure of the crack-tip constraint and even the in-plane T_{11} exhibits three-dimensional (3D) effects [2-4], the 3D T -stresses solutions were obtained by running finite element analyses (FEA) for 3PB specimens with wide ranges of the crack depth-to-width ratio ($a/W = 0.2$ to 0.8) and the specimen thickness-to-width ratio ($B/W = 0.1$ to 40). The 2D T_{11} solutions have been provided for 3PB specimen in many numerical studies [5-10].

The results show that the 3D T_{11} at the specimen mid-plane tended to deviate from the 2D T_{11} as B/W increased, with the deviation saturating for $B/W \geq 2$. The mid-plane 3D T_{11} for $B/W = 0.1$ to 40 was high as 54% when $a/W = 0.2$, suggesting that 3D effects should be properly considered for cases of short crack length, especially when T_{11} is negative. The mid-plane T_{33} increased with B/W and was close to the plane strain value σ_{11} for $B/W \geq 2$.

2. *T*-stress

In an isotropic linear elastic body containing a crack subjected to symmetric (mode I) loading, the Williams series expansion [11] of the 3D stress components near the crack tip field can be written as [3]

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \begin{Bmatrix} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \\ 2\nu \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} T_{11} \\ 0 \\ T_{33} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (1)$$

where r and θ are the in-plane polar coordinates of the plane normal to the crack front shown in Fig. 1, K_I is the local mode I stress intensity factor (SIF) and ν is Poisson's ratio. Here, x_1 is the direction formed by the intersection of the plane normal to the crack front and the plane tangential to the crack plane. T_{11} and T_{33} are the amplitudes of the second-order terms in the three-dimensional series expansions of the crack front stress field in the x_1 and x_3 directions, respectively.

Different methods have been applied to compute the elastic *T*-stress for test specimens, as summarized by Sherry et al. [10]. In this study, an efficient finite element method developed by Nakamura and Parks [11] based on an interaction integral was used to determine the elastic *T*-stresses.

The crack tip T_{11} -stress on the crack front is related to the interaction integral by

$$T_{11} = \frac{E}{1-\nu^2} \left\{ \frac{I}{F} + \nu \varepsilon_{33} \right\} \quad (2)$$

where E is Young's modulus, ν is Poisson's ratio and ε_3 identifies the out-of-plane strain at the crack tip in the direction tangential to the crack front. I represents the interaction integral, and F indicates the unit magnitude ($F = 1$).

Once the T_{11} -stress is obtained, the T_{33} -stress can be obtained using the following relationship:

$$T_{33} = E\varepsilon_{33} + \nu T_{11} \quad (3)$$

More details of this method can be found in Nakamura and Parks [11] and Qu and Wang [12].

3. Finite Element Analysis (FEA)

3.1 Description of the finite element model

In the present study, 3D elastic FEA was conducted to calculate the elastic *T*-stresses (T_{11} and T_{33}) for a 3PB test specimen with a straight crack. Fig. 2 shows a sketch of the loads and geometry. In this figure, a , B , W and S are the crack length and the specimen thickness, width and support span,

respectively. For all current calculations, the specimen width was set as $W = 25$ mm, with a support span of $S = 4W$.

To systematically quantify the out-of-plane crack-tip constraint effect of the 3PB specimen, the thickness-to-width ratios $B/W = 0.1, 0.25, 0.5, 1, 1.5, 2$ and 40 were considered to cover the B/W range studied experimentally by Rathbun et al. [1]. For each B/W , the crack depth-to-width ratios $a/W = 0.2, 0.3, 0.4, 0.45, 0.5, 0.55, 0.6, 0.7$ and 0.8 were considered to investigate the in-plane constraint.

The material is assumed to be linearly elastic (isotropic and homogeneous). Young's modulus $E = 206$ GPa and Poisson's ratio $\nu = 0.3$ were set based on ferritic steel, which is the most widely used material in engineering. 3D finite elements were used to build a one-quarter symmetric model of the 3PB specimen, as shown in Fig. 3(a). The finite element model used 20-noded isoparametric 3D solid elements with reduced $(2 \times 2 \times 2)$ Gauss integration. Sixteen singular elements were used around the crack tip for all cases in this study. Twenty equivalent rows of meshes were spaced inside the crack tube with radius $R_s = 0.4$ mm (Fig. 3(b)). In the present FEA models, 365740 to 393194 nodes with 86912 to 93840 elements were used, and the details for the generated mesh are summarized in the Appendix.

WARP3D [13] was used as the FEA solver. The load set for the elastic FEA corresponded to the 2D SIF $K_0 = 1$ MPa m^{1/2} calculated from the following equation from the ASTM standard [14].

$$K = \frac{PS}{BW^{3/2}} f(a/W) \quad (4)$$

where f is a function of a/W and is defined in the standard.

3.2 T -stresses for 3PB specimens

T_{11} was evaluated as the average of the values of T_{11} obtained from domain 2 to domain 20. Good independence of the T value on the choice of domain was obtained, as the differences in the T -stress results from domain 2 to domain 20 were within 1% of one another, except for the values in the vicinity of the free surface. The obtained mid-plane T_{11} and T_{33} stresses are summarized in Tables 1 and 2, respectively, in the normalized form of $\beta_k = T_{kk}(\mathbf{x})^{1/2}/K_0$ ($k = 1$ or 3). The T -stresses at the specimen mid-plane received special attention because fracture initiation occurs at this location (e.g., [1, 2]).

First, the obtained mid-plane β_1 values were compared with the 2D β_1 solutions obtained by different authors [5-9] as a validity check. Sherry et al. reported that these 2D solutions varied significantly [10] and compiled them as a polynomial function of a/W . However, in this work, Kfoury's plane-strain solutions [6] were chosen for comparison with our 3D β_1 solutions based on the expectation that the 3D β_1 will approach the plane-strain values, as shown in Fig. 4(a). The mid-plane β_1 exhibited 3D effects and monotonously decreased with increasing B/W but saturated to

values very close to the plane-strain solutions, as shown in Fig. 4(b). This tendency was similar to that observed by Nakamura and Parks for a single edge-cracked plate under pure bending [3].

Another finding was that β_1 was a monotonously increasing function of a/W , regardless of B/W . The results showed that negative β_1 , and thus loss of the in-plane crack-tip constraint, was anticipated for cases of $a/W \leq 0.3$.

Fig. 5 shows the mid-plane β_3 solutions for various thicknesses and crack depths. In Fig. 5(a), it is observed that β_3 is a monotonously increasing function of B/W , as expected. The bounding value of β_3 for each a/W was close to the plane strain value β_1 , and a relative thickness of $B/W = 40$ was sufficient for β_3 to saturate to the bounding value, as shown in Fig. 5(b).

β_3 for the ASTM standard 3PB specimen [14], for which $B/W = 0.5$ and $0.45 \leq a/W \leq 0.55$, was negative. This finding seemed to support the fact that J_c was not bounded in the case of increasing B/W for 3PB specimens [1].

Interestingly, in Fig. 5(b), β_3 was not always a monotonously increasing function of a/W , as observed for the thin specimens of $B/W = 0.1, 0.25$ and 0.5 . For example, β_3 for $B/W = 0.1$ was a monotonously decreasing function of a/W and thus might lead to the incorrect conclusion that deep cracks lose the out-of-plane crack-tip constraint. However, by normalizing T_{33} in terms of $T_{33}(\mathcal{H})^{1/2}/K_0$ (W was constant for all cases in this study) as shown in Fig. 6, it is clearly seen that T_{33} increased monotonously as a/W increased for all B/W s, which means that the out-of-plane crack-tip constraint level was strengthened due to the increase in crack depth, although the increase rate was smaller than $a^{1/2}$.

4. Discussion

In addition to the mid-plane T -stresses, the variations of the β_1 and β_3 solutions in the thickness direction were also plotted for various thicknesses for $a/W = 0.5$ in Fig. 7 and 8, respectively. Note that the mid-side node values were omitted in this figure. As observed in the left part of Fig. 7, the in-plane β_1 distributions changed little overall compared with the mid-plane value in the range of $x_3/(B/2) = 0$ to 0.8 . Specifically, these differences were in the range of 4.1 to 15.3% . The differences were less than 5% if $x_3/(B/2)$ was in the range of 0 to 0.5 , regardless of B/W .

On the other hand, the out-of-plane β_3 distributions in Fig. 8 showed a visible decrease in the thickness direction, considering that the ordinate of this figure ranges from -14 to 2 . However, the rate of decrease became small as B/W increased, as is clear for the case of $B/W = 40$. Note that both T_{11} and T_{33} diverged significantly in the vicinity of the free surface ($x_3/(B/2) = 0.8$ to 1.0) because ξ_3 tends to be singular near the free surface and is not well calculated using FEA [3, 4]. Thus, the T -stresses near the free surface calculated by the present FEA method are known to be unreliable [12] and require further study.

5. Summary

In the present study, the T -stress solutions for 3PB specimens with a wide range of the crack depth-to-width ratio ($a/W = 0.2$ to 0.8) and the specimen thickness-to-width ratio ($B/W = 0.1$ to 40) were calculated using 3D elastic FEA. The results showed that 3D T_{11} at the specimen mid-plane tended to deviate from the 2D T_{11} as B/W increased, with the deviation saturating for $B/W \geq 2$. The mid-plane 3D T_{11} between cases of $B/W = 0.1$ and 40 was large as 54% for $a/W = 0.2$ and suggested that the 3D effects should be properly considered for cases of short crack length, especially when T_{11} is negative. The mid-plane T_{33} increased with B/W and was close to the plane strain value σ_{11} for $B/W \geq 2$.

Acknowledgments

This work was supported in part by JSPS KAKENHI Grant Number 24561038. Their support is greatly appreciated.

Appendix

List of figures

Fig. 1 Three-dimensional coordinate system for the region along the crack front

Fig. 2. Sketch of the loads and geometry of the 3PB specimens

Fig. 3. Typical finite element model of a 3PB specimen ($W = 25$ mm, $S/W = 4$, $a/W = 0.5$, $B/W = 0.5$)

Fig. 4 Normalized T_{11} solutions (β_1) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)

Fig. 5 Normalized T_{33} solutions (β_3) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)

Fig. 6 Normalized T_{33} solutions ($T_{33}(W)^{1/2}/K_0$) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)

Fig. 7 Variations of β_1 in the thickness direction along the crack front for various thicknesses when $a/W = 0.5$ ($\nu = 0.3$)

Fig. 8 Variations of β_3 in the thickness direction along the crack front for various thicknesses when $a/W = 0.5$ ($\nu = 0.3$)

References

- [1] Rathbun HJ, Odette GR, Yamamoto T, Lucas GE. Influence of statistical and constraint loss size effects on cleavage fracture toughness in the transition-A single variable experiment and database. *Engineering Fracture Mechanics*. 2006;73:134-58.
- [2] Meshii T, Lu K, Takamura R. A failure criterion to explain the test specimen thickness effect on fracture toughness in the transition temperature region. *Engineering Fracture Mechanics*. 2013;104: 184-197.
- [3] Nakamura T, Parks DM. Determination of elastic T -stress along three-dimensional crack fronts using an interaction integral. *International Journal of Solids and Structures*. 1992;29(13):1597-1611.
- [4] Fernández-Canteli A, Giner E, Fernández-Sáez J, Fernández-Zúñiga. A unified analysis of the in-plane and out-of-plane constraints in 3-D linear elastic fracture toughness, *Proceedings of the 19th European Conference on Fracture*. Kazan, Russia. 2012; p.1-8.
- [5] Leever PS, Radon JC. Inherent stress biaxiality in various fracture specimen geometries. *International Journal of Fracture*. 1982;19:311-325.
- [6] Kfoury AP. Some evaluations of the elastic T -term using Eshelby's method. *International Journal of Fracture*. 1986;30(4):301-315.

- [7] Cardew GE, Goldthorpe MR, Howard IC, Kfoury AP. On the Elastic T -term. In: Bibly BA, Miller KJ, Willis JR, editors. Fundamentals of Deformation and Fracture. Cambridge: Cambridge University Press; 1984. p. 465-476.
- [8] Fett T. T -stresses in rectangular plates and circular disks. Engineering Fracture Mechanics. 1998;60(5-6):631-652.
- [9] Yang B, Ravi-Chandar K. Evaluation of elastic T -stress by the stress difference method. Engineering Fracture Mechanics. 1999;64(5):589-605.
- [10] Sherry AH, Moran B, Nakamura T. Compendium of T -stress solutions for two and three dimensional cracked geometries. Fatigue and Fracture of Engineering Materials and Structures. 1995;18:141-155.
- [11] Williams ML. On the stress distribution at the base of a stationary crack. Journal of Applied Mechanics. 1957;24:111-114.
- [12] Qu J, Wang X. Solutions of T -stresses for quarter-elliptical corner cracks in finite thickness plates subject to tension and bending. International Journal of Pressure Vessels and Piping. 2006;83(8):593-606.
- [13] Gullerud A, Koppenhoefer K, Roy Y, RoyChowdhury S, Walters M, Bichon B, et al. WARP3D Release 15 Manual. Civil Engineering, Report No UIUCENG-95-2012, University of Illinois at Urbana-Champaign. 2004.
- [14] ASTM. E1921-10 Standard test method for determination of reference temperature, T_0 , for ferritic steels in the transition range. Annual Book of ASTM Standards. Philadelphia PA: American Society for Testing and Materials; 2010.

Table 1. Normalized T_{11} solutions (β_1) at the specimen mid-plane for 3PB specimens ($\nu=0.3$).

a/W									
B/W	0.2	0.3	0.4	0.4	0.5	0.5	0.6	0.7	0.8
				5		5			
0.1	- 0.158	- 0.022	0.11 5	0.18 5	0.25 8	0.33 4	0.41 5	0.60 5	0.87 6
0.2 5	- 0.182	- 0.055	0.07 9	0.14 9	0.22 1	0.29 7	0.37 9	0.57 6	0.86 6
0.5	- 0.188	- 0.059	0.07 3	0.14 3	0.21 6	0.29 4	0.37 9	0.57 4	0.82 0
1	- 0.220	- 0.078	0.06 3	0.13 3	0.20 4	0.27 5	0.34 8	0.50 8	0.72 2
1.5	- 0.236	- 0.107	0.02 8	0.09 5	0.16 3	0.23 1	0.30 2	0.46 4	0.69 6
2	- 0.240	- 0.122	0.00 5	0.07 0	0.13 6	0.20 5	0.27 7	0.44 7	0.68 8
40	- 0.244	- 0.125	0.00 01	0.06 5	0.13 3	0.20 4	0.26 7	0.44 0	0.68 4

Table 2 Normalized T_{33} solutions (β_3) at the specimen mid-plane for 3PB specimens ($\nu=0.3$)

a/W									
B/W	0.2	0.3	0.4	0.4	0.5	0.5	0.6	0.7	0.8
0.1	- 0.861	- 1.041	- 1.184	- 1.245	- 1.301	- 1.351	- 1.395	- 1.461	- 1.458
0.2	- 0.530	- 0.617	- 0.682	- 0.708	- 0.728	- 0.742	- 0.749	- 0.725	- 0.587
0.5	- 0.353	- 0.388	- 0.404	- 0.405	- 0.399	- 0.386	- 0.362	- 0.262	- 0.041
1	- 0.246	- 0.206	- 0.168	- 0.144	- 0.115	- 0.079	- 0.036	0.07 3	0.20 5
1.5	- 0.216	- 0.142	- 0.072	- 0.038	- 0.003	0.03 3	0.07 0	0.14 7	0.23 0
2	- 0.197	- 0.118	- 0.040	- 0.004	0.03 1	0.065	0.09 7	0.15 9	0.23 2
40	- 0.059	- 0.024	0.01 1	0.03 0	0.04 9	0.06 9	0.11 4	0.16 6	0.23 6

Table A.1 Summary of the generated mesh ($W = 25$ mm, $S/W = 4$, $R_s = 0.4$ mm)

B/W	a/W	0.2	0.3	0.4	0.4	0.5	0.5	0.6	0.7	0.8
					5		5			
0.1	Δ/a	4.0	2.7	2.0	1.8	1.6	1.5	1.3	1.1	1.0
	CD	12	16	20	26	30	34	40	48	56
	NR	61	53	45	41	37	33	29	21	13
	na (na_bias)	189 (2)								
0.25	Δ/a	4.0	2.7	2.0	1.8	1.6	1.5	1.3	1.1	1.0
	CD	12	16	20	26	30	34	40	48	56
	NR	61	53	45	41	37	33	29	21	13
	na (na_bias)	189 (2)								
0.5	Δ/a	4.0	2.7	2.0	1.8	1.6	1.5	1.3	1.1	1.0
	CD	12	16	20	26	30	34	40	48	56
	NR	61	53	45	41	37	33	29	21	13
	na (na_bias)	18 9 (2)								
1	Δ/a	4.0	2.7	2.0	1.8	1.6	1.5	1.3	1.1	1.0
	CD	12	16	20	26	30	34	40	48	56

	NR	61	53	45	41	37	33	29	21	13
	na (na_bias)	189 (2)								
1.5	<i>Ma</i>	4.0	2.7	2.0	1.8	1.6	1.5	1.3	1.1	1.0
	CD	12	16	20	26	30	34	40	48	56
	NR	61	53	45	41	37	33	29	21	13
	na (na_bias)	189 (2)								
2	<i>Ma</i>	4.0	2.7	2.0	1.8	1.6	1.5	1.3	1.1	1.0
	CD	12	16	20	26	30	34	40	48	56
	NR	61	53	45	41	37	33	29	21	13
	na (na_bias)	189 (2)								
40	<i>Ma</i>	4.0	2.7	2.0	1.8	1.6	1.5	1.3	1.1	1.0
	CD	12	15	24	20	20	24	30	40	45
	NR	50	50	36	36	36	32	28	20	12
	na (na_bias)	18 9 (2)	24 3(1)							

Highlights

- T -stress solutions 3PB specimens with various crack depths and thicknesses were obtained.
- Mid-plane T_{11} and T_{33} were reported for 3PB specimens with $a/W = 0.2 \sim 0.8$ and $B/W = 0.1 \sim 40$.
- T_{11} showed 3D effect, and approached 2D plane strain solutions for large thickness.
- T_{33} increased with thickness, and saturated to $\sqrt{3}\sigma_y$ for $B/W \geq 2$.

Fig. 1 Three-dimensional coordinate system for the region along the crack front

Fig. 2 Sketch of the loads and geometry of the 3PB specimens

2

Fig. 3 Typical finite element model of a 3PB specimen
($W = 25$ mm, $S/W = 4$, $a/W = 0.5$, $B/W = 0.5$)

3

Fig. 4 Normalized T_{11} solutions (β_{11}) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)

4

Fig. 5 Normalized T_{33} solutions (β_{33}) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)

5

Fig. 6 Normalized T_{33} solutions ($T_{33}(\pi W)^{1/2}/K_0$) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)

6

Fig. 7 Variations of β_{11} in the thickness direction along the crack front for various thicknesses when $a/W = 0.5$ ($\nu = 0.3$)

7

Fig. 8 Variations of β_{33} in the thickness direction along the crack front for various thicknesses when $a/W = 0.5$ ($\nu = 0.3$)

8