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Three-dimensional $T$-stresses for three-point-bend specimens with large thickness variation

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Abstract

Three-point-bend (3PB) test specimens are useful for the systematic investigation of the influence of statistical and constraint loss size effects on the cleavage fracture toughness of a material in the ductile-to-brittle transition temperature range. Because the in- and out-of-plane elastic $T$-stresses ($T_{11}$ and $T_{33}$) are a measure of the crack-tip constraint and even the in-plane $T_{11}$ exhibits three-dimensional (3D) effects, the 3D $T$-stresses solutions were obtained by running finite element analyses (FEA) for 3PB specimens with wide ranges of the crack depth-to-width ratio ($a/W = 0.2$ to 0.8) and the specimen thickness-to-width ratio ($B/W = 0.1$ to 40). The results show that the 3D $T_{11}$ at the specimen mid-plane tended to deviate from the 2D $T_{11}$ as $B/W$ increased, with the deviation saturating for $B/W \geq 2$. The mid-plane $T_{33}$ increased with $B/W$ and was close to the plane strain value $T_{11}$ for $B/W \geq 2$.

Keywords: Elastic $T$-stress, Three-point-bend specimen, Finite element analysis, Fracture toughness, Constraint effect

Nomenclature

- $B$: Specimen thickness
- $E$: Young’s modulus
- $F$: Unit magnitude (see Eq. (2))
- $I$: Interaction integral
- $K_1$: Local mode I stress intensity factor (SIF)
- $K_0$: 2D SIF for elastic analysis
- $R_s$: Crack tube radius
- $S$: Support span for 3PB specimen
- $T_{11}$, $T_{33}$: $T$-stresses
$W$ Specimen width
$a$ Crack length
$r, \theta$ In-plane polar coordinates
$x_j$ Crack-tip local coordinates ($j = 1, 2, 3$)
$\Delta$ Singular element size
$\beta_1, \beta_3$ Normalized $T$-stresses
$\varepsilon_3$ Out-of-plane strain
$\nu$ Poisson’s ratio
$\sigma$ Stress components ($i, j = 1, 2, 3$)

1. Introduction

Three-point-bend (3PB) test specimens are useful for the systematic investigation of the statistical and constraint loss size effects on the cleavage fracture toughness of a material in the ductile-to-brittle transition temperature range [1, 2]. Because the in-plane and out-of-plane $T$-stresses ($T_{11}$ and $T_{33}$) are a measure of the crack-tip constraint and even the in-plane $T_{11}$ exhibits three-dimensional (3D) effects [2-4], the 3D $T$-stresses solutions were obtained by running finite element analyses (FEA) for 3PB specimens with wide ranges of the crack depth-to-width ratio ($a/W = 0.2$ to 0.8) and the specimen thickness-to-width ratio ($B/W = 0.1$ to 40). The 2D $T_{11}$ solutions have been provided for 3PB specimen in many numerical studies [5-10].

The results show that the 3D $T_{11}$ at the specimen mid-plane tended to deviate from the 2D $T_{11}$ as $B/W$ increased, with the deviation saturating for $B/W \geq 2$. The mid-plane 3D $T_{11}$ for $B/W = 0.1$ to 40 was high as 54% when $a/W = 0.2$, suggesting that 3D effects should be properly considered for cases of short crack length, especially when $T_{11}$ is negative. The mid-plane $T_{33}$ increased with $B/W$ and was close to the plane strain value $T_{33}$ for $B/W \geq 2$. 
2. T-stress

In an isotropic linear elastic body containing a crack subjected to symmetric (mode I) loading, the Williams series expansion [11] of the 3D stress components near the crack tip field can be written as

\[
\begin{pmatrix}
\sigma_{11} \\
\sigma_{22} \\
\tau_{12} \\
\tau_{23} \\
\tau_{33}
\end{pmatrix} = \frac{K_I}{\sqrt{2\pi}r} \begin{pmatrix}
\cos \frac{\theta}{2} & 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \\
\cos \frac{\theta}{2} & 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \\
2\nu \cos \frac{\theta}{2} & \theta \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \\
\theta \sin \frac{\theta}{2} \cos \frac{3\theta}{2} & 0 \\
0 & 0
\end{pmatrix} \begin{pmatrix}
T_{11} \\
T_{33}
\end{pmatrix} + \begin{pmatrix}
\epsilon_{33} \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\]

where \(r\) and \(\theta\) are the in-plane polar coordinates of the plane normal to the crack front shown in Fig. 1, \(K_I\) is the local mode I stress intensity factor (SIF) and \(\nu\) is Poisson’s ratio. Here, \(x_1\) is the direction formed by the intersection of the plane normal to the crack front and the plane tangential to the crack plane. \(T_{11}\) and \(T_{33}\) are the amplitudes of the second-order terms in the three-dimensional series expansions of the crack front stress field in the \(x_1\) and \(x_3\) directions, respectively.

Different methods have been applied to compute the elastic T-stress for test specimens, as summarized by Sherry et al. [10]. In this study, an efficient finite element method developed by Nakamura and Parks [11] based on an interaction integral was used to determine the elastic T-stresses.

The crack tip \(T_{11}\)-stress on the crack front is related to the interaction integral by

\[
T_{11} = \frac{E}{1 - \nu^2} \left\{ \frac{I}{F} + \nu \epsilon_{33} \right\}
\]

where \(E\) is Young’s modulus, \(\nu\) is Poisson’s ratio and \(\epsilon_{33}\) identifies the out-of-plane strain at the crack tip in the direction tangential to the crack front. \(I\) represents the interaction integral, and \(F\) indicates the unit magnitude (\(F = 1\)).

Once the \(T_{11}\)-stress is obtained, the \(T_{33}\)-stress can be obtained using the following relationship:

\[
T_{33} = E\epsilon_{33} + \nu T_{11}
\]

More details of this method can be found in Nakamura and Parks [11] and Qu and Wang [12].

3. Finite Element Analysis (FEA)

3.1 Description of the finite element model

In the present study, 3D elastic FEA was conducted to calculate the elastic T-stresses \((T_{11}\) and \(T_{33}\)) for a 3PB test specimen with a straight crack. Fig. 2 shows a sketch of the loads and geometry. In this figure, \(a, B, W\) and \(S\) are the crack length and the specimen thickness, width and support span,
respectively. For all current calculations, the specimen width was set as \( W = 25 \, \text{mm} \), with a support span of \( S = 4W \).

To systematically quantify the out-of-plane crack-tip constraint effect of the 3PB specimen, the thickness-to-width ratios \( B/W = 0.1, 0.25, 0.5, 1, 1.5, 2 \) and 4 were considered to cover the \( B/W \) range studied experimentally by Rathbun et al. [1]. For each \( B/W \), the crack depth-to-width ratios \( a/W = 0.2, 0.3, 0.4, 0.45, 0.5, 0.55, 0.6, 0.7 \) and 0.8 were considered to investigate the in-plane constraint.

The material is assumed to be linearly elastic (isotropic and homogeneous). Young’s modulus \( E = 206 \, \text{GPa} \) and Poisson’s ratio \( \nu = 0.3 \) were set based on ferritic steel, which is the most widely used material in engineering. 3D finite elements were used to build a one-quarter symmetric model of the 3PB specimen, as shown in Fig. 3(a). The finite element model used 20-noded isoparametric 3D solid elements with reduced \((2\times2\times2)\) Gauss integration. Sixteen singular elements were used around the crack tip for all cases in this study. Twenty equivalent rows of meshes were spaced inside the crack tube with radius \( R_s = 0.4 \, \text{mm} \) (Fig. 3(b)). In the present FEA models, 365740 to 393194 nodes with 86912 to 93840 elements were used, and the details for the generated mesh are summarized in the Appendix.

WARP3D [13] was used as the FEA solver. The load set for the elastic FEA corresponded to the 2D SIF \( K_0 = 1 \, \text{MPa} \, \text{m}^{1/2} \) calculated from the following equation from the ASTM standard [14].

\[
K = \frac{PS}{BW^{3/2}} f(a/W) \quad (4)
\]

where \( f \) is a function of \( a/W \) and is defined in the standard.

3.2 \( T \)-stresses for 3PB specimens

\( T_{11} \) was evaluated as the average of the values of \( T_{11} \) obtained from domain 2 to domain 20. Good independence of the \( T \) value on the choice of domain was obtained, as the differences in the \( T \)-stress results from domain 2 to domain 20 were within 1% of one another, except for the values in the vicinity of the free surface. The obtained mid-plane \( T_{11} \) and \( T_{33} \) stresses are summarized in Tables 1 and 2, respectively, in the normalized form of \( \beta_{kk} = T_{kk}(\pi a/W)^{1/2}/K_0 \) \((k = 1 \text{ or } 3)\). The \( T \)-stresses at the specimen mid-plane received special attention because fracture initiation occurs at this location (e.g., [1, 2]).

First, the obtained mid-plane \( \beta_{kk} \) values were compared with the 2D \( \beta_{kk} \) solutions obtained by different authors [5-9] as a validity check. Sherry et al. reported that these 2D solutions varied significantly [10] and compiled them as a polynomial function of \( a/W \). However, in this work, Kfouri’s plane-strain solutions [6] were chosen for comparison with our 3D \( \beta_{kk} \) solutions based on the expectation that the 3D \( \beta_{kk} \) will approach the plane-strain values, as shown in Fig. 4(a). The mid-plane \( \beta_{kk} \) exhibited 3D effects and monotonously decreased with increasing \( B/W \) but saturated to
values very close to the plane-strain solutions, as shown in Fig. 4(b). This tendency was similar to that observed by Nakamura and Parks for a single edge-cracked plate under pure bending [3].

Another finding was that \( \beta_1 \) was a monotonously increasing function of \( a/W \), regardless of \( B/W \). The results showed that negative \( \beta_1 \), and thus loss of the in-plane crack-tip constraint, was anticipated for cases of \( a/W \leq 0.3 \).

Fig. 5 shows the mid-plane \( \beta_3 \) solutions for various thicknesses and crack depths. In Fig. 5(a), it is observed that \( \beta_3 \) is a monotonously increasing function of \( B/W \), as expected. The bounding value of \( \beta_3 \) for each \( a/W \) was close to the plane strain value \( \beta_3 \), and a relative thickness of \( B/W = 40 \) was sufficient for \( \beta_3 \) to saturate to the bounding value, as shown in Fig. 5(b).

\( \beta_3 \) for the ASTM standard 3PB specimen [14], for which \( B/W = 0.5 \) and \( 0.45 \leq a/W \leq 0.55 \), was negative. This finding seemed to support the fact that \( J_c \) was not bounded in the case of increasing \( B/W \) for 3PB specimens [1].

Interestingly, in Fig. 5(b), \( \beta_3 \) was not always a monotonously increasing function of \( a/W \), as observed for the thin specimens of \( B/W = 0.1, 0.25 \) and 0.5. For example, \( \beta_3 \) for \( B/W = 0.1 \) was a monotonously decreasing function of \( a/W \) and thus might lead to the incorrect conclusion that deep cracks lose the out-of-plane crack-tip constraint. However, by normalizing \( T_{33} \) in terms of \( T_{33}(W)^{1/2}/K_0 \) (\( W \) was constant for all cases in this study) as shown in Fig. 6, it is clearly seen that \( T_{33} \) increased monotonously as \( a/W \) increased for all \( B/Ws \), which means that the out-of-plane crack-tip constraint level was strengthened due to the increase in crack depth, although the increase rate was smaller than \( a^{1/2} \).

4. Discussion

In addition to the mid-plane \( T \)-stresses, the variations of the \( \beta_1 \) and \( \beta_3 \) solutions in the thickness direction were also plotted for various thicknesses for \( a/W = 0.5 \) in Fig. 7 and 8, respectively. Note that the mid-side node values were omitted in this figure. As observed in the left part of Fig. 7, the in-plane \( \beta_1 \) distributions changed little overall compared with the mid-plane value in the range of \( x_3/(B/2) = 0 \) to 0.8. Specifically, these differences were in the range of 4.1 to 15.3\%. The differences were less than 5\% if \( x_3/(B/2) \) was in the range of 0 to 0.5, regardless of \( B/W \).

On the other hand, the out-of-plane \( \beta_3 \) distributions in Fig. 8 showed a visible decrease in the thickness direction, considering that the ordinate of this figure ranges from -14 to 2. However, the rate of decrease became small as \( B/W \) increased, as is clear for the case of \( B/W = 40 \). Note that both \( T_{11} \) and \( T_{33} \) diverged significantly in the vicinity of the free surface (\( x_3/(B/2) = 0.8 \) to 1.0) because \( \varepsilon_3 \) tends to be singular near the free surface and is not well calculated using FEA [3, 4]. Thus, the \( T \)-stresses near the free surface calculated by the present FEA method are known to be unreliable [12] and require further study.
5. Summary
In the present study, the $T$-stress solutions for 3PB specimens with a wide range of the crack depth-to-width ratio ($a/W = 0.2$ to 0.8) and the specimen thickness-to-width ratio ($B/W = 0.1$ to 40) were calculated using 3D elastic FEA. The results showed that 3D $T_{11}$ at the specimen mid-plane tended to deviate from the 2D $T_{11}$ as $B/W$ increased, with the deviation saturating for $B/W \geq 2$. The mid-plane 3D $T_{11}$ between cases of $B/W = 0.1$ and 40 was large as 54% for $a/W = 0.2$ and suggested that the 3D effects should be properly considered for cases of short crack length, especially when $T_{11}$ is negative. The mid-plane $T_{33}$ increased with $B/W$ and was close to the plane strain value $T_{11}$ for $B/W \geq 2$.

Acknowledgments
This work was supported in part by JSPS KAKENHI Grant Number 24561038. Their support is greatly appreciated.

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References


Table 1. Normalized $T_{11}$ solutions ($\beta$) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$).

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Table 2 Normalized $T_{33}$ solutions ($\beta_3$) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)

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Highlights

・ T-stress solutions 3PB specimens with various crack depths and thicknesses were obtained.
・ Mid-plane $T_{11}$ and $T_{33}$ were reported for 3PB specimens with $a/W = 0.2 \sim 0.8$ and $B/W = 0.1 \sim 40$.
・ $T_{11}$ showed 3D effect, and approached 2D plane strain solutions for large thickness.
・ $T_{33}$ increased with thickness, and saturated to $\nu T_{11}$ for $B/W \geq 2$.

![Three-dimensional coordinate system for the region along the crack front](image.png)

Fig. 1 Three-dimensional coordinate system for the region along the crack front
Fig. 2 Sketch of the loads and geometry of the 3PB specimens

Fig. 3 Typical finite element model of a 3PB specimen

$(W = 25 \text{ mm}, S/W = 4, a/W = 0.5, B/W = 0.5)$
Fig. 4 Normalized $T_{11}$ solutions ($\beta_{11}$) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$) 

Fig. 5 Normalized $T_{33}$ solutions ($\beta_{33}$) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)
Fig. 6 Normalized $T_{33}$ solutions ($T_{33}(\pi W)^{1/2}/K_0$) at the specimen mid-plane for 3PB specimens ($\nu = 0.3$)

Fig. 7 Variations of $\beta_{11}$ in the thickness direction along the crack front for various thicknesses when $a/W = 0.5$ ($\nu = 0.3$)
Fig. 8 Variations of $\beta_{33}$ in the thickness direction along the crack front for various thicknesses when $a/W = 0.5$ ($\nu = 0.3$)