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Summary The paper proposes a method in finite element analysis for estimating natural frequencies of a disk tensioned by rolling, without the use of eigenvalue analysis. The natural frequencies of a disk vary when the localized plastic deformation caused by roll-tensioning induces residual stresses. Tensioning is used for improving the dynamic stability of circular saws; the optimal condition of rolling can be predicted from natural frequency characteristics. In the proposed method, the natural frequencies after rolling are easily estimated from the mode shapes of the disk before rolling and the stress distribution after rolling. The method is based on ideas similar to thermal stress and sensitivity analysis rather than on eigenvalue analysis. The effectiveness of the method is shown by comparing the natural frequency characteristics obtained by this method with those by eigenvalue analysis.

Key words vibration, roll-tensioning, plastic deformation, eigenfrequency, FEM

1 Introduction

Circular saws are widely used for cutting and forming wood. Since large transverse displacements by resonance occasionally occur in cutting works, a process called "tensioning has been traditionally used for improving the dynamic stability. It is reported in [1, 2] that the resonance is caused by the reduction of natural frequencies of the sawblade due to the thermal compressive stress induced by cutting heat in the peripheral region. It is desired, therefore, that the tensile stress should be previously induced by tensioning, so that the natural frequency concerning critical speed instability would be enhanced. Especially, the process using localized plastic deformation by rollers, called roll-tensioning, and the automation of this process are expected to improve the performance similarly to the skill and experience of craftsmen.

Some studies on roll-tensioning have been done in [3], where residual stresses of disks induced by roll-tensioning were analyzed and compared with experimental results. In [4, 5] natural frequencies of a disk tensioned by rollers as well as residual stresses were obtained. These studies contained theoretical analyses for uniform thin disks, however, in practical sawblades, there are some holes against heat, slots and tips at the periphery. It is not easy to develop analytical theories for such sawblades. Contrary to this, sawblades with slots and tips can be easily modeled within the FEM (Finite Element Method). In order to determine the optimal rolling condition, it is important to grasp their relation to natural frequency characteristics. The use of eigenvalue analysis for that purpose is accompanied by much computational effort.

In this paper, we propose a method using FEM for easy estimates of natural frequencies of disks tensioned by rolling, which is based on an idea similar to the sensitivity analysis. The

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variation in stiffness due to rolling is analyzed in a way of temperature loading as shown in [3], instead of the static stress analysis. Natural frequency characteristics for various rolling and clamping positions by a flange are obtained by the proposed method, and it is shown that there exists an optimal position of rolling. The effectiveness of the proposed method is ascertained by comparison with results obtained by eigenvalue analysis. The role of tensioning is assessed from the change in natural frequencies and mode shapes.

2
Tensioning of circular sawblades by rollers

In the tensioning process, a sawblade is compressed within a certain annular contact zone between two opposing rollers as shown in Fig. 1. The sawblade is plastically deformed in the rolling region as shown in Fig. 2, [3], and there occur tensile residual stresses in the inner and outer regions. Especially at the outer edge, the tensile residual stresses become large. The tensile stresses overcome thermal compressive stresses induced during cutting, and result in enhancing natural frequencies. Generally, it is known that the critical speed instability in rotating disks occurs when the backward-traveling wave frequency is equal to zero, [3, 4]. In the case of circular saws, the thermal stresses also affect the stability, and it is necessary to enhance natural frequencies by tensioning.

As reported in [6], vibration characteristics are improved or not improved according to the stress distribution within the sawblade induced by rolling, even if there occur residual tensile stresses at the periphery. The residual stress distribution is affected by roller path position, roller load, number of rolling processes and so on. The roller path position greatly affects the residual stress distribution, and it is an important problem to determine the optimal rolling position, [7]. However, it is difficult to determine the optimal position from stress analysis alone. We investigate the optimal rolling position by vibration analysis, using FEM and a method for estimating the change in natural frequencies.

3
Analysis of the problem

We explain the method using FEM for analyzing and estimating vibration characteristics of thin circular disks tensioned by rolling.

3.1
Equation of motion

In general, the equation of motion of a disk modeled by FEM is described by

$$[M]\{\ddot{u}\} + [K]\{u\} = \{f\} \ , \tag{1}$$

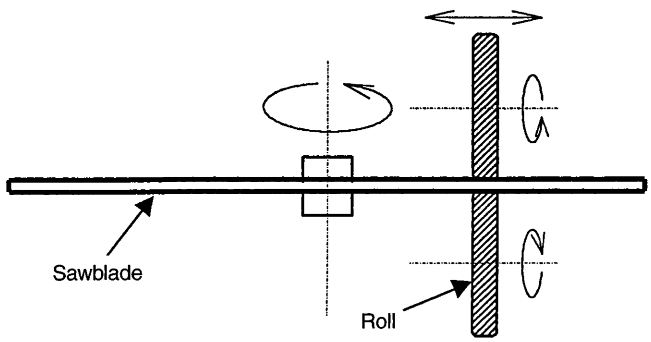

Fig. 1. Roll tensioning of a circular sawblade

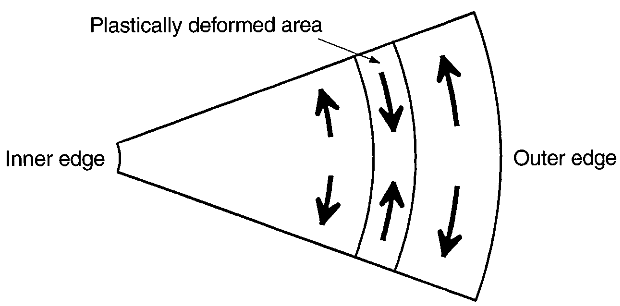

Fig. 2. Typical distribution of residual stresses induced by the plastic deformation at rolling

where $[M]$ and $[K]$ are mass and stiffness matrices; $\{u\}$ and $\{f\}$ are deflection and external force vectors, respectively. Natural frequencies of disks can be changed by roll-tensioning, which induces a certain pattern of residual stresses by the localized plastic deformation. The mass distribution is not changed due to rolling, but the stiffness characteristics vary greatly. Let the variation in stiffness from $[K]$ be $[K^{(\sigma)}]$. Then, the equation of motion of a disk after rolling is described by

$$[M]\{\ddot{\tilde{u}}\} + ([K] + [K^{(\sigma)}])\{\tilde{u}\} = \{f\} \ , \tag{2}$$

where $\{\tilde{u}\}$ denotes the deflection of a disk after rolling. In the case of shell elements and plane stress, the elements of the matrix $[K^{(\sigma)}]$ are given by

$$[K_i^{(\sigma)}] = \int_V [B_d]^T \begin{bmatrix} \sigma_x & & & & & \\ \tau_{xy} & \sigma_y & & & \text{sym.} & \\ 0 & 0 & \sigma_x & & & \\ 0 & 0 & \tau_{xy} & \sigma_y & & \\ 0 & 0 & 0 & 0 & \sigma_x & \\ 0 & 0 & 0 & 0 & \tau_{xy} & \sigma_y \end{bmatrix} [B_d] \mathrm{d}V \ , \tag{3}$$

where σ_x and σ_y are normal stresses in the x and y directions in the element coordinates and τ_{xy} is the shearing stress; $[B_d]$ is a matrix whose components consist of derivatives of deflection. The symbol T means the transpose of a matrix, and V is the volume.

3.2
Stress analysis for $[K^{(\sigma)}]$

In order to calculate $[K^{(\sigma)}]$, we must know the stress distribution of a disk after rolling. For obtaining the stress distribution, we adopt a similar thermal stress analysis as in Ref. [3], without directly analyzing plastic deformation by rolling. Namely, a temperature distribution load, which is considered to induce the same stress distribution as that by plastic deformation, is used for the analysis. It is assumed that the plastic deformation induced by rolling has the maximum value at the center of rolling region and decreases toward the peripheral region rapidly and continuously. We use the temperature distribution of the Gaussian function type

$$T(r) = A \exp\left\{ -\frac{(r - r_c)^2}{2s^2} \right\} \ , \tag{4}$$

where $T(r)$ is the temperature at the radius r, A is the maximum temperature, r_c is the radius at the center of the temperature distribution, and s is the standard deviation related to the width of the distribution determined from the roller load, the number and the width of rolling. The distribution of 95% is included within the extent of $4s$ that is considered to be an appropriate rolling width, [3].

This analytical method has the advantage of less computational effort than a method directly analyzing the plastic deformation. The effectiveness of this method is shown by the comparison of both analytical results and measurements concerning residual stresses,[3], and natural frequencies, [4], of tensioned disks.

3.3
Vibration analysis of the variation in natural frequencies

Natural frequencies of a disk with a stress distribution can be obtained from the following eigenvalue problem, after calculating $[K^{(\sigma)}]$ by the thermal stress analysis,

$$([K] + [K^{(\sigma)}] - \tilde{\omega}^2[M])\{\tilde{\phi}\} = \{0\} \ , \tag{5}$$

where $\tilde{\omega}$ and $\{\tilde{\phi}\}$ with the upper tilde $\sim$ denote the natural frequency and the mode shape vector of a disk after rolling, respectively. To obtain natural frequency curves for the center radius r_c, it is necessary to determine the optimal rolling radius. A large amount of eigenvalue analysis is required for that.

We propose, therefore, a method for easy prediction of natural frequencies after rolling similar to the sensitivity analysis rather than the eigenvalue analysis. Let ω_i and $\{\phi_i\}$ be the i-th natural angular frequency and the mode shape vector of a disk before rolling, respectively. These satisfy the following relation:

$$([K] - \omega_i^2[M])\{\phi_i\} = \{0\} \ . \tag{6}$$

Considering r_c as a design variable, the partial derivative of Eq. (6) with respect to r_c produces

$$([K] - \omega_i^2[M])\frac{\partial\{\phi_i\}}{\partial r_c} + \left(\frac{\partial[K]}{\partial r_c} - \omega_i^2\frac{\partial[M]}{\partial r_c}\right)\{\phi_i\} - \frac{\partial\omega_i^2}{\partial r_c}[M]\{\phi_i\} = \{0\} \ . \tag{7}$$

Multiplying Eq. (7) by $\{\phi_i\}^T$ from the left side, and considering the symmetry of $[M]$ and $[K]$, we obtain the sensitivity of the eigenvalues (squared frequencies) for r_c

$$\frac{\partial\omega_i^2}{\partial r_c} = \frac{\{\phi_i\}^T\left(\frac{\partial[K]}{\partial r_c} - \omega_i^2\frac{\partial[M]}{\partial r_c}\right)\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}} \ . \tag{8}$$

Since for a disk after rolling the matrix $[M]$ is not changed, but $[K]$ is changed by $[K^{(\sigma)}]$, the variation in the i-th eigenvalue for arbitrary r_c is given by

$$\Delta\omega_i^2(r_c) = \frac{\{\phi_i\}^T[K^{(\sigma)}]\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}} \ . \tag{9}$$

Equation (9) means that once the mode vector before rolling $\{\phi_i\}$ is obtained, natural frequencies after rolling can be easily predicted without eigenvalue analysis.

Equation (9) can also be derived in another way. We assume that the mode vector after rolling $\{\tilde{\phi}\}$ is described by superposition of mode vectors before rolling as

$$\{\tilde{\phi}\} = [\Phi]\{\xi\} = [\{\phi_1\}\cdots\{\phi_n\}]\{\xi\} \ , \tag{10}$$

where $\{\xi\}$ is the modal coordinate vector. Substituting Eq. (10) into Eq. (5) and multiplying the resulting equation by $[\Phi]^T$ from the left side, we obtain

$$(\lceil k \rfloor + [\Phi]^T[K^{(\sigma)}][\Phi] - \tilde{\omega}^2\lceil m \rfloor)\{\xi\} = \{0\}; \tag{11}$$
$$\lceil m \rfloor = [\Phi]^T[M][\Phi], \quad \lceil k \rfloor = [\Phi]^T[K][\Phi] \ .$$

Here $\lceil m \rfloor$ and $\lceil k \rfloor$ are diagonal matrices whose elements consist of modal masses $m_i\,(i = 1, \ldots, n)$ and modal stiffnesses k_i, respectively. If $[\Phi]^T[K^{(\sigma)}][\Phi]$ also becomes a diagonal matrix, the left-hand side of Eq. (11) becomes a system of uncoupled equations, whose i-th equation is expressed by

$$(k_i + \{\phi_i\}^T[K^{(\sigma)}]\{\phi_i\} - \tilde{\omega}_i^2 m_i)\xi_i = 0 \ . \tag{12}$$

Then, looking for natural frequencies after rolling we obtain

$$\tilde{\omega}_i^2 = \omega_i^2 + \frac{\{\phi_i\}^T[K^{(\sigma)}]\{\phi_i\}}{m_i} \ . \tag{13}$$

It is found from $m_i = \{\phi_i\}^T[M]\{\phi_i\}$ that the second term in the right-hand side of Eq. (13), which means the variation from $\omega_i^2\ (=k_i/m_i)$ before rolling, is equal to Eq. (9) and the natural frequency characteristic after rolling can be predicted from Eq. (9).

Next, we consider the prediction accuracy of Eq. (9) or (13) in relation to diagonal $[\Phi]^T[K^{(\sigma)}][\Phi]$. Let $[\Phi]$ and $[\tilde{\Phi}]$ be the modal matrix before and after rolling, respectively. The matrix $[\Phi]^T[K^{(\sigma)}][\Phi]$ becomes a diagonal matrix when vibration modes before and after rolling are not changed regardless of stress distribution, that it, $[\tilde{\Phi}] = [\Phi]$ holds. This condition is introduced as follows: since vibration modes after rolling have orthogonality with respect to the stiffness matrix $[K]$ + $[K^{(\sigma)}]$, the relation

$$[\tilde{\Phi}]^T([K] + [K^{(\sigma)}])[\tilde{\Phi}] = \lceil \tilde{k} \rfloor \ , \tag{14}$$

holds. If $[\tilde{\Phi}] = [\Phi]$ holds, the following relation is also satisfied:

$$\lceil k \rfloor + [\Phi]^T [K^{(\sigma)}][\Phi] = \lceil \tilde{k} \rfloor \ , \tag{15}$$

Since $\lceil k \rfloor$ and $\lceil \tilde{k} \rfloor$ are diagonal matrices, $[\Phi]^T[K^{(\sigma)}][\Phi]$ must also be diagonal under the condition that $[\tilde{\Phi}] = [\Phi]$. Therefore, it is expected that the prediction of natural frequencies by Eq. (9) has high accuracy when there is little difference between mode shapes before and after rolling, because then $[\Phi]^T[K^{(\sigma)}][\Phi]$ is approximately diagonal. Later we calculate and ascertain whether or not

$$[\hat{K}^{(\sigma)}] = [\Phi]^T [K^{(\sigma)}][\Phi] \ , \tag{16}$$

is approximately a diagonal matrix. Concerning the condition that $[\tilde{\Phi}] = [\Phi]$, we ascertain by checking the orthogonality of the following matrix with respect to mass matrix:

$$[\hat{m}] = [\tilde{\Phi}]^T [M][\Phi] \ . \tag{17}$$

When $[\Phi]$ and $[\tilde{\Phi}]$ are normalized so that modal masses become unity, $[\hat{m}]$ will be approximately a unit matrix.

4
Numerical examples and considerations

First, we obtain the variation in natural frequencies for various r_c by eigenvalue analysis and show that there exists an optimal rolling radius. Next, we compare the variation in natural frequencies by Eq. (9) with that by eigenvalue analysis. A sawblade is modeled by a circular disk of the outer diameter 255 mm, as shown in Fig. 3. The disk is analyzed by FEM (the software ANSYS) using four-node plane shell elements.

4.1
Stress distribution

We consider the case where rolling is performed around the center radius 65 mm. Figure 4 shows the temperature distribution $T(r)$ which is assumed to induce an equivalent stress distribution to that by rolling. Parameters of Eq. (4) are r_c = 65 mm, A = 50°C and s = 5.

Figure 5 shows a stress distribution within the disk induced by temperature distribution loading shown in Fig. 4, where only 1/12 part of the disk is drawn. It is found that large circumferential compressive stresses are recognized within the region of $T(r)$ loading, and there appear circumferential tensile stresses.

4.2
Natural frequency characteristics by eigenvalue analysis

We investigate the effect r_c in $T(r)$ on natural frequencies after rolling $\tilde{\omega}_i$. Even if the temperature loading is the same as in Fig. 4, $\tilde{\omega}_i$ varies with r_c and there exists the optimal rolling position for enhancing $\tilde{\omega}_i$. Since a practical circular sawblade is clamped by a flange, the clamping radius r_f affects the optimal rolling position, and then $\tilde{\omega}_i$ curves are calculated for r_c and r_f. The disk for analysis is assumed to be perfectly fixed in the region of $r \leq r_f$. The outer edge is free. Regarding the relationship between tensioning and the number of nodal diameters of vibration modes N_d, it is reported in [6] that compressive stresses during cutting make natural frequencies of $N_d = 0$ and 1 increase, but those of $N_d \geq 2$ decrease. Therefore, it becomes necessary to enhance natural frequencies of $N_d \geq 2$ beforehand, and appropriate positions for rolling should be selected so that natural frequencies of $N_d \geq 2$ become large.

Figure 6 shows $\tilde{\omega}_i$ curves of $N_d = 0$ to 4 calculated by FEM when r_c is varied from 25 mm to 105 mm by 10 mm, with keeping the temperature distribution in Fig. 4. Figure 6a–c present the results for r_f = 25 mm, 55 mm and 75 mm, respectively. For reference, natural frequencies before rolling ω_i are marked at r_c = 0. It is seen that the tendency of curves of N_d = 2 to 4 differs from that of N_d = 0 and 1. For example, in Fig. 6a, natural frequencies of N_d = 2 to 4 increase with r_c until they reach the maximum values near r_c = 65 mm; then they decrease. They become smaller than ω_i for r_c near the outer edge, and the effect of rolling is not

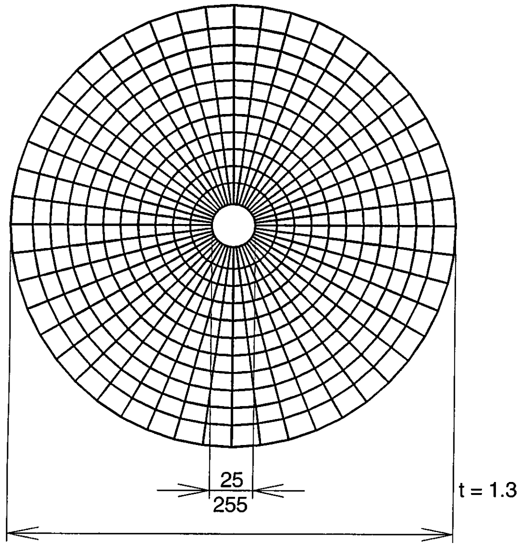

Fig. 3. FEM model for the disk

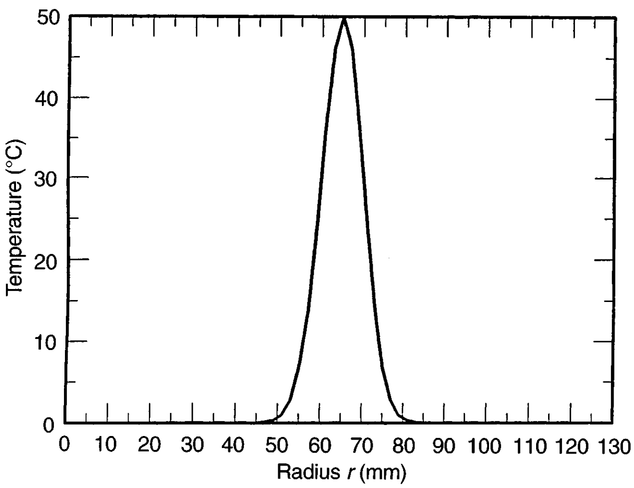

Fig. 4. Temperature distribution assumed for residual stresses induced by rolling

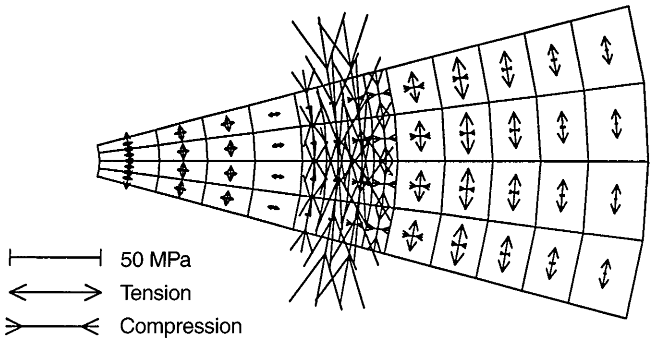

Fig. 5. Stress distribution in the disk subjected to the temperature distribution shown in Fig. 4

recognized. Contrary to this, natural frequencies of $N_d = 0$ and 1 reach the minimum values near $r_c = 45$ mm; then they increase. In Fig. 6b and c, curves of $N_d = 2$ are not much changed for r_c. Comparing Fig. 6a–c, such r_c, for which $\tilde{\omega}_i$ of $N_d = 3$ and 4 becomes maximal, also increases for larger r_f. It is found that the optimal rolling position should be determined from $\tilde{\omega}_i$ curves for r_c and r_f.

Next, we consider the optimal rolling position from the viewpoint of sensitivity of natural frequencies in relation to the difference of tendency between curves of $N_d = 3$ and 4 and curves of $N_d = 0$ and 1. The reason why natural frequencies are varied due to rolling is that stiffness characteristics within the disk are locally changed by the stress distribution. In order to investigate the effect of stiffness characteristics on the change in natural frequencies, a disk is

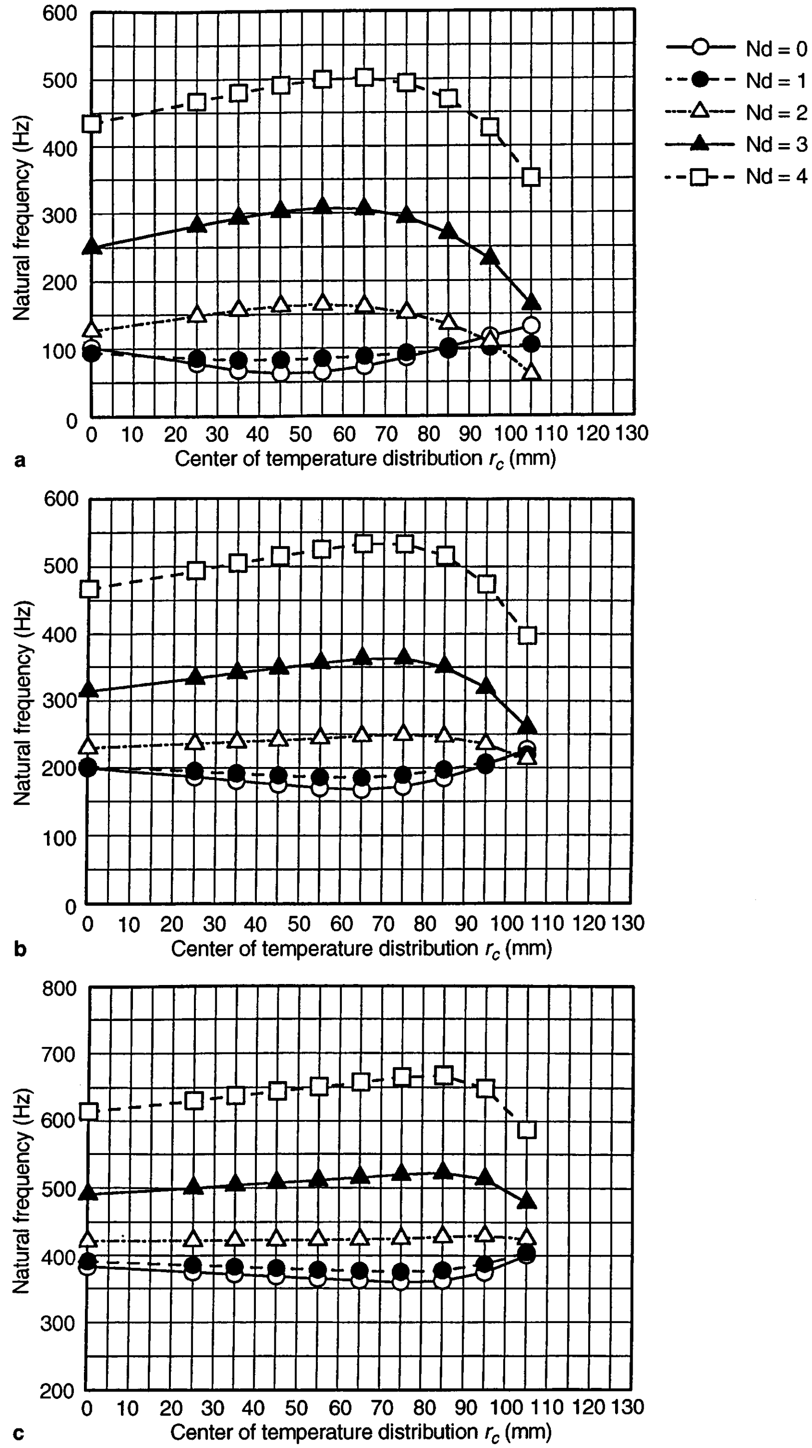

Fig. 6a–c. Relation between natural frequencies and r_c. **a** $r_f = 25$ mm, **b** $r_f = 55$ mm, **c** $r_f = 75$ mm

analyzed whose stiffness for the annular zone of the width 10 mm centered at the radius r_a is enhanced by 1%. The relationship between the sensitivity of natural frequencies and r_a is shown in Fig. 7 for $r_f = 25$ mm. The sensitivity for $N_d = 0$ and 1 monotonously decreases toward the outer edge, whereas the sensitivity for $N_d = 3$ and 4 substantially increases. Remember the results by $T(r)$ loading as shown in Fig. 5. Large compressive stresses occur near r_c, and tensile stresses occur in the inner and outer regions, where the absolute values of compressive stresses are quite larger than tensile stresses. When r_c is selected to be near the outer edge, natural frequencies of $N_d = 3$ and 4 decrease due to the compressive stresses as shown in Fig. 6a because the sensitivity for $N_d = 3$ and 4 is high.

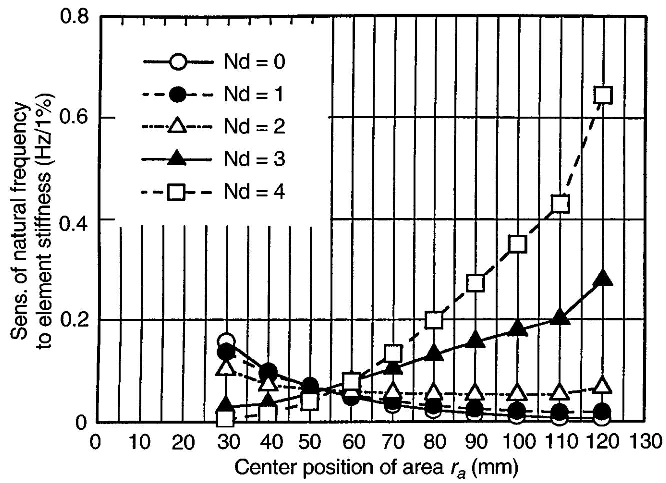

Fig. 7. Sensitivity of natural frequency with respect to stiffness ($r_f = 25$ mm)

4.3
Verification of estimation of natural frequencies by Eq. (9)

Figure 8 shows values of $\Delta\omega_i = \tilde{\omega}_i - \omega_i$ estimated by Eq. (9) for r_c = 25 mm to 105 mm by 10 mm. Figures 8a–c present the results for $r_f = 25$ mm, 55 mm and 75 mm, respectively, which correspond to Fig. 6a–c by eigenvalue analysis. Comparing them we observe, that natural frequencies of N_d = 2 to 4 become maximal for such values of r_c which well coincide with each other except for N_d = 2 in Fig. 8c. We compare values for N_d = 4 for $r_f = 25$ mm, at which large variation takes place. In Fig. 8a, $\Delta\omega$ = 70.0 Hz at r_c = 65 mm, whereas in Fig. 6a $\Delta\omega$ = 67.6 Hz from $\tilde{\omega} = 502.1$ Hz and ω = 434.5 Hz. The difference between both values is 2.4 Hz and small enough. Accordingly, it is possible to predict the variation in natural frequencies by Eq. (9) with high accuracy.

Next, we check two matrices of Eqs. (16) and (17) concerning the accuracy of Eq. (9). Table 1 shows the components of $[\hat{K}^{(\sigma)}]$ with respect to N_d = 0 to 4 for r_c = 65 mm and $r_f = 25$ mm. Most of nondiagonal components are zero. There appear nonzero components between N_d = 0 and 3 and between N_d = 1 and 4, but these values are quite smaller than the diagonal components. Therefore, $[\hat{K}^{(\sigma)}]$ is approximately diagonal, and it is considered that Eq. (9) has a high accuracy. Table 2 shows components of $[\hat{m}]$ in Eq. (17) for $r_f = 25$ mm and $r_c = 65$ mm. All the nondiagonal components are zero, and the diagonal ones, except for N_d = 3, become unity. Since $[\hat{m}]$ is approximately a unit matrix, the condition that $[\tilde{\Phi}] = [\Phi]$ also holds. Accordingly, it is concluded that natural frequencies are greatly varied due to roll-tensioning, but the mode shapes change little.

5
Conclusions

A method in finite element analysis for estimating natural frequencies of disks tensioned by rolling without the use of eigenvalue analysis has been proposed. Instead of static stress analysis, the temperature distribution load assumed for plastic deformation was used for calculating the variation in stiffness due to rolling. The summary of results is as follows:

(1) It is ascertained from verification by Eqs. (16) and (17) and comparison with results by eigenvalue analysis that the change in natural frequencies can be predicted from Eq. (9)

Table 1. Matrix $[\hat{K}^{(\sigma)}]$ for r_c = 65 mm and r_f = 25 mm

Nd	0	1	2	3	4
0	-1.70×10^5	0.00	0.00	0.00	−6.26
1	0.00	-3.41×10^4	0.00	−4.35	0.00
2	0.00	0.00	3.82×10^5	0.00	0.00
3	0.00	−4.35	0.00	1.16×10^6	0.00
4	−6.26	0.00	0.00	0.00	2.40×10^6

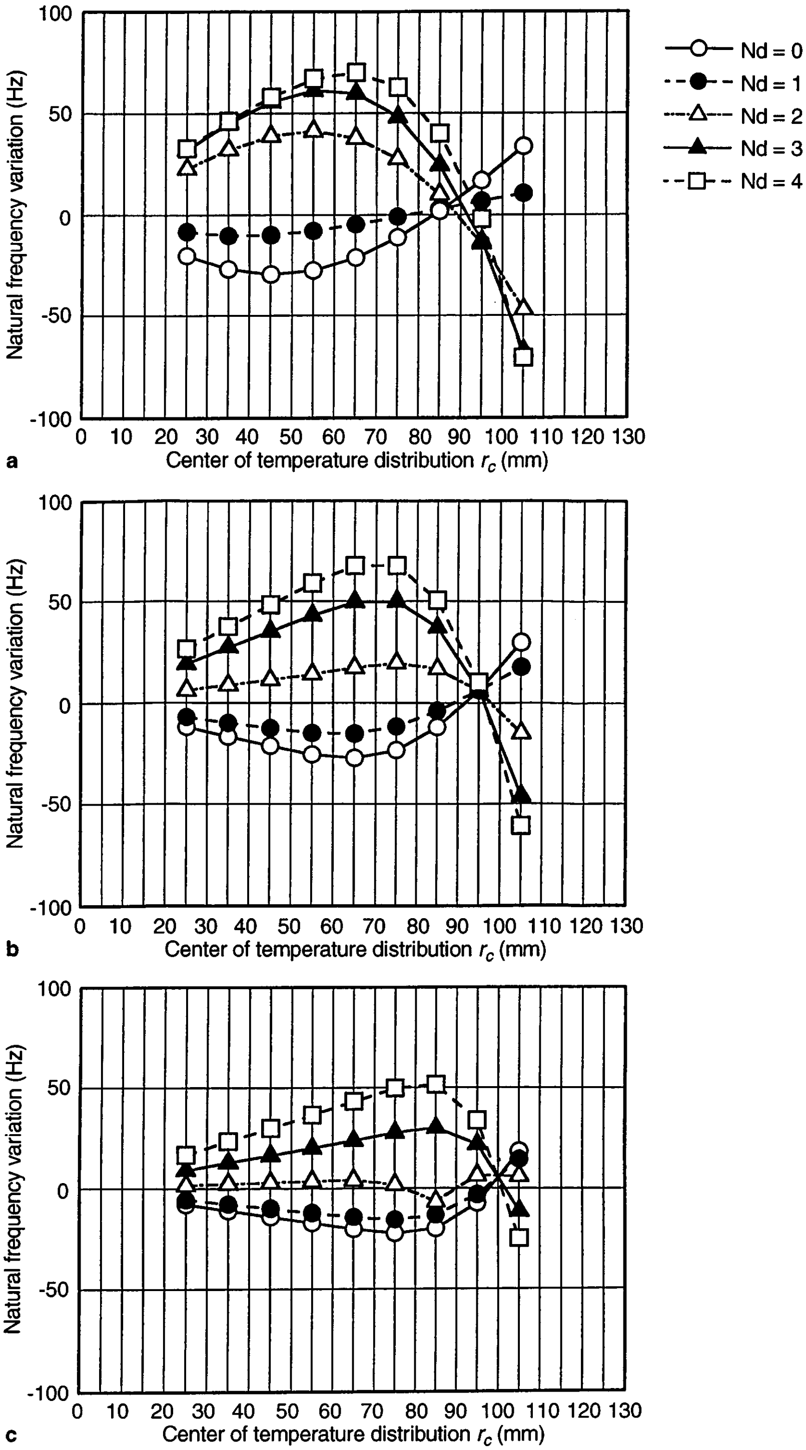

Fig. 8a–c. Variation in natural frequencies for r_c by Eq. (9). **a** $r_f = 25$ mm, **b** $r_f = 55$ mm, **c** $r_f = 75$ mm

Table 2. Matrix $[\hat{m}]$ for r_c = 65 mm and r_f = 25 mm

mode shape after rolling \ mode shape before rolling	Nd	0	1	2	3	4
	0	1.00	0.00	0.00	0.00	0.00
	1	0.00	1.00	0.00	0.00	0.00
	2	0.00	0.00	1.00	0.00	0.00
	3	0.00	0.00	0.00	0.93	0.00
	4	0.00	0.00	0.00	0.00	1.00

with the advantage of less computational effort and high accuracy. It is found that natural frequencies are greatly varied due to rolling, but the mode shapes change little.

(2) For certain rolling radii r_c and clamping radii r_f, natural frequencies of $N_d \geq 2$ become smaller than those before rolling, and the purpose of tensioning cannot be achieved. Therefore, it is necessary to determine the optimal rolling position from natural frequency characteristics calculated for various r_c and r_f.

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