Closed Form Stress Intensity Factor for Arbitrarily Located Inner-Circumferential Surface Crack in a Cylinder Subjected To Axisymmetric Bending Loads

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CLOSED FORM STRESS INTENSITY FACTOR FOR AN ARBITRARILY LOCATED INNER CIRCUMFERENTIAL SURFACE CRACK IN A CYLINDER SUBJECTED TO AXISYMMETRIC BENDING LOADS

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ABSTRACT

In this paper, the stress intensity factor for a inner circumferential surface crack in a hollow cylindrical shell under axisymmetric bending loads is studied. The closed form equations of stress intensity factor and inclination angles at the cylinder edges were derived. These equations can appropriately evaluate the effects of cylinder length and crack location on the stress intensity factor. The validity of these equations were illustrated by comparing solutions with numerical ones. The results showed that the stress intensity factor increases as the cylinder length decreases, and as the crack gets near the cylinder edge.

KEY WORDS : Fracture Mechanics, Stress Intensity Factor, Cylindrical Shell, Axisymmetric Loads, Circumferential Crack, Compliance, Finite Length, Crack Location.

1. INTRODUCTION

The stress intensity factor (SIF) for a cylindrical structure such as a pressure vessel or a pipe which contains a part-through crack seems to be difficult to solve analytically, even for a fundamental one such as a circumferential crack. In the past, SIF for this crack configuration in a finite-sized cylinder were evaluated numerically by using the technique of the finite elements[1], or, analytically under restricted condition of a long cylinder[2, 3]. These methods will provide useful results for confined cases, but are not suitable to study the effects of cylinder configuration on the SIF parametrically. To meet this need, the authors recently developed a simplified but an analytical method to evaluate the SIF for a inner surface circumferential crack in a hollow cylindrical shell subjected to axisymmetric loads[4], which can take account of the effects that the cylinder length and the crack location have on the SIF. The numerical examples of the method showed that the SIF of this configuration subjected to axisymmetric bending loads increases as the cylinder length decreases, and as the crack gets near the cylinder edge. As these facts cannot be read from the well known handbooks[5], the authors were encouraged to derive more direct equations to evaluate the SIF under typical loading conditions to meet the practical need in the field of design and inspection. In addition, formulation of the inclination angles at the edge of a cylinder was felt necessary for our further study, in which the edge constraint will have to be taken into consideration in the problem of thermal stress.

In the following chapters, closed form equations of SIF and of inclination angle at the cylinder edge will be derived by applying the method proposed in our previous paper [4]. Two cases, that is, the case where the bending loads of the same value are applied on the both ends of the cylinder, and the case for a given crack face traction equal to the normal stress acting on the crack plane when the body is uncracked, will be considered.

After formulation, the validity of the closed form equation of SIF will be illustrated by comparing solutions with those by FEM. At the same time, the effects that the cylinder length and the crack location have on these SIF will be demonstrated.

2. DERIVATION OF CLOSED FORM SIF AND INCLINATION ANGLE

In this chapter, the method developed by the authors [4] will be introduced first, as a minimum information to derive the closed form equations of SIF and of inclination angle at the cylinder edge. After that, closed form equations of SIF and inclination angle at the edge of a cylinder are derived concretely for two most fundamental circumferential crack problems by applying the method.

2. 1 The SIF for a circumferential crack in a cylinder under axisymmetric loads

In our previous paper, a simplified method to evaluate the SIF for a case shown in Figure 1(a) was developed by modeling the cylinder with spring-connected beams on an elastic foundation shown in Figure 1(b) [4]. Here, P_1, P_2, M_1 and M_2 are defined as values per unit length in the circumferential direction for cylinders, and as values per unit thickness for the beams, respectively.

In this case, the flexural rigidity of the prismatic beam was $D = EW^3/12(1-\nu^2)$, and the spring constant of the elastic foundation k was given by the following equation,

$$k = \frac{EW}{R_m^2} = 4\beta^4 D \ ; \ \beta^4 = \frac{EW}{4R_m^2 D} \tag{1}$$

where, R_m : mean radius, W: thickness, E : Young' s Modulus, ν : Poisson's Ratio. P_1 and P_2 were supposed to be loaded in the direction coinciding with one of the principal axes of the prismatic bar. In the same manner, the direction of the vectors M_1 and M_2 was assumed to coincide with the other principal axis. Note that $1/\beta$ has a dimension of length.

The required SIF K_M was evaluated as the SIF of a single edge cracked strip under pure bending moment M_C, as

$$K_M = \frac{M_C}{Z}\sqrt{\pi a} \cdot F_M(\xi = a/W) \tag{2}$$

where $Z = W^2/6$ is the section modulus, F_M is the correction factor for finite width under pure bending, and M_C is the moment at the rotary spring in Figure 1(b), obtained as is shown next.

Let's denote the shearing force, bending moment, deflection and inclination angles of upper and lower ends at the rotary spring as F_C, M_C, y_C and θ_{C1}, θ_{C2}, respectively. Then,

$$\mathbf{F}_{Cg} = \begin{bmatrix} F_C & M_C & y_C & \theta_{C1} & \theta_{C2} \end{bmatrix}^t \tag{3}$$

is derived by the following equation, which gives M_C in Eq. (2).

$$\mathbf{F}_{Cg} = \mathbf{C}_g^{-1} \times \mathbf{B}_g \times \mathbf{P}_{ABg} \tag{4}$$

$$\mathbf{C}_g = \begin{bmatrix} \lambda_{yP}^*(h_1,0) & \lambda_{yM}^*(h_1,0) & -1 & 0 & 0 \\ \lambda_{\theta P}^*(h_1,0) & \lambda_{\theta M}^*(h_1,0) & 0 & -1 & 0 \\ -\lambda_{yP}(0,h_2) & \lambda_{yM}(0,h_2) & -1 & 0 & 0 \\ -\lambda_{\theta P}(0,h_2) & \lambda_{\theta M}(0,h_2) & 0 & 0 & -1 \\ 0 & -2\Delta\lambda & 0 & 1 & -1 \end{bmatrix} \tag{5}$$

$$\mathbf{B}_g = \begin{bmatrix} -\lambda_{yP}(h_1,0) & -\lambda_{yM}(h_1,0) & 0 & 0 \\ -\lambda_{\theta P}(h_1,0) & -\lambda_{\theta M}(h_1,0) & 0 & 0 \\ 0 & 0 & -\lambda^*_{yP}(0,h_2) & -\lambda^*_{yM}(0,h_2) \\ 0 & 0 & -\lambda^*_{\theta P}(0,h_2) & -\lambda^*_{\theta M}(0,h_2) \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

$$\mathbf{P}_{ABg} = \begin{bmatrix} P_1 & M_1 & P_2 & M_2 \end{bmatrix}' \quad (7)$$

The various λ and λ^* in Eqs. (5) and (6) are compliances whose respective definition can be referred in APPENDIX. The compliance of the spring is evaluated by $\Delta\lambda$; the infinite beam's increment in compliance due to the presence of a crack. In the following sections, closed form equations of SIF and of inclination angle at the cylinder edge will be derived for specific problems, based on the method described here.

2. 2 Formulation of the SIF and the inclination angles for the case of axisymmetric bending load pair on both ends

In this section, the case where a pair of axisymmetric bending load is applied on both ends (Figure 2) will be focused.

This case can be treated as a case in which the loads in Figure 1 are $P_1 = P_2 = 0$ and $M_1 = M_2 = M$. By substituting these loads in Eq. (7), and by calculating the various compliances in Eqs. (5) and (6) by referring APPENDIX 1, the shearing force F_C and moment M_C at the spring are obtained from Eq. (4). They can be written in a closed form as follows.

$$\begin{aligned} \frac{F_C}{M} = & -4\beta(\sin\beta h_1 \sinh\beta h_2 - \sin\beta h_2 \sinh\beta h_1) \\ & \times \{\sin\beta H - \sinh\beta H \\ & \quad + \beta D\cdot\Delta\lambda[\cos\beta H - \cosh\beta H - \cos\beta(h_1-h_2) + \cosh\beta(h_1-h_2)]\} \\ & / \ [2 - \cos 2\beta H - \cosh 2\beta H \\ & \quad + \beta D\cdot\Delta\lambda\{\sin 2\beta H - \sinh 2\beta H + 2(\sinh 2\beta h_1 - \sin 2\beta h_1) + 2(\sinh 2\beta h_2 - \sin 2\beta h_2) \\ & \quad\quad + \sin 2\beta h_1 \cosh 2\beta h_2 - \cos 2\beta h_1 \sinh 2\beta h_2 + \sin 2\beta h_2 \cosh 2\beta h_1 - \cos 2\beta h_2 \sinh 2\beta h_1\}] \end{aligned} \quad (8)$$

$$\frac{M_C}{M} = \tag{9}$$
$$\{2(\cos\beta h_1\cosh\beta h_1+\cos\beta h_2\cosh\beta h_2)$$
$$+\sin\beta(H+h_1)\sinh\beta h_2-\sin\beta h_2\sinh\beta(H+h_1)+\sin\beta(H+h_2)\sinh\beta h_1-\sin\beta h_1\sinh\beta(H+h_2)$$
$$-\cos\beta(H+h_1)\cosh\beta h_2-\cos\beta h_2\cosh\beta(H+h_1)-\cos\beta(H+h_2)\cosh\beta h_1-\cos\beta h_1\cosh\beta(H+h_2)\}$$
$$/\ [2-\cos 2\beta H-\cosh 2\beta H+$$
$$\beta D\cdot\Delta\lambda\{\sin 2\beta H-\sinh 2\beta H+2(\sinh 2\beta h_1-\sin 2\beta h_1)+2(\sinh 2\beta h_2-\sin 2\beta h_2)$$
$$+\sin 2\beta h_1\cosh 2\beta h_2-\cos 2\beta h_1\sinh 2\beta h_2+\sin 2\beta h_2\cosh 2\beta h_1-\cos 2\beta h_2\sinh 2\beta h_1\}]$$

By using the moment M_C, the required SIF K_M can be calculated as the SIF of a single edge cracked strip under pure bending moment M_C,

$$K_M=\frac{M_C}{Z}\sqrt{\pi a}\cdot F_M(\xi)=\left(\frac{M_C}{M}\right)\cdot\left[\frac{M}{Z}\sqrt{\pi a}\cdot F_M(\xi)\right] \tag{10}$$
$$\equiv\left(\frac{M_C}{M}\right)\cdot K_{Mbeam}$$

where K_{Mbeam} represents the SIF of a single edge cracked strip under pure bending moment M. Note that F_C/M and M_C/M are values that are completely determined just by the various compliances of the two beams and that of the spring.

The inclination angle θ_A, θ_B at the edge A, B can be derived by treating these edges as an end point of a beam on an elastic foundation shown in APPENDIX Figure A. 1. The angle θ_A is obtained by substituting $h = h_1$, $x = 0$, $x' = h_1$, $P_1 = 0$, $M_1 = M$, $P_2 = F_C$, $M_2 = M_C$ in Eq. (A 1), and θ_B by setting $h = h_2$, $x = h_2$, $x' = 0$, $P_1 = (-F_C)$, $M_1 = M_C$, $P_2 = 0$, $M_2 = M$ in Eq. (A 1).

$$\vec{\theta}=\begin{bmatrix}\theta_A/M\\ \theta_B/M\end{bmatrix}=\mathbf{q}+\mathbf{Q}\cdot\begin{bmatrix}F_C/M\\ M_C/M\end{bmatrix} \tag{11}$$

Here the matrixes **q** and **Q** are defined as follows to simplify the expression.

$$\mathbf{q}=\begin{bmatrix}\lambda_{\theta M}(0,h_1)\\ \lambda^*_{\theta M}(h_2,0)\end{bmatrix}\ ;\quad \mathbf{Q}=\begin{bmatrix}\lambda^*_{\theta P}(0,h_1) & \lambda^*_{\theta M}(0,h_1)\\ -\lambda_{\theta P}(h_2,0) & \lambda_{\theta M}(h_2,0)\end{bmatrix} \tag{12}$$

2. 3 Formulation of the SIF and the inclination angles for the case of a given crack face traction

It is often conducted, based on the principle of superposition, to evaluate the SIF for a case subjected to external loads, by analyzing the case with a given crack face traction equal to the normal stress acting on the crack plane when the body is uncracked. In this section, the case in which the stress distribution is linear, and equal to M_0 in a term of moment as shown in Figure 3, will be focused.

Suppose that the uncracked cylinder is subjected to external bending load pair M on both edges, and M_0 is a resultant moment acting on the crack plane when the cylinder is uncracked. In this case, by rewriting the results which Hetényi [6] derived for a beam on an elastic foundation to those for a cylindrical shell, M_0 is formulated as a function of crack location and M.

$$M_0 = \phi_a \cdot M; \quad \phi_a = \frac{\sinh \beta h_1 \cos \beta h_2 + \cosh \beta h_1 \sin \beta h_2 + \sinh \beta h_2 \cos \beta h_1 + \cosh \beta h_2 \sin \beta h_1}{\sinh \beta H + \sin \beta H} \tag{13}$$

By applying the principle of superposition, the required SIF for the case shown in Figure 3 can be derived by substituting (M_0/ϕ_a) for M in Eq. (10),

$$\begin{aligned} K_{M0} &= \frac{M_C}{Z}\sqrt{\pi a}\cdot F_M(\xi) = \left(\frac{M_C}{M}\right)\cdot\frac{1}{\phi_a}\cdot\left[\frac{M_0}{Z}\sqrt{\pi a}\cdot F_M(\xi)\right] \\ &\equiv \left(\frac{M_C}{M}\right)\cdot\frac{1}{\phi_a}\cdot K_{M0beam} \end{aligned} \tag{14}$$

where K_{M0beam} represents the SIF of a single edge cracked strip under pure bending moment M_0.

The inclination angles at the edges $\Delta\theta_1$ and $\Delta\theta_2$ can be derived as the increment due to the presence of the inner circumferential surface crack in a cylinder, under bending loads (M_0/ϕ_a) on both ends. By introducing the compliance $\lambda_{cyl\,0}$, which is the compliance of a uncracked cylinder subjected to a pair of bending load M on both ends, and by using Eq. (11), the required inclination angles are given as follows.

$$\begin{bmatrix} \Delta\theta_1 / M_0 \\ \Delta\theta_2 / M_0 \end{bmatrix} = \frac{1}{\phi_a}\cdot\left\{\mathbf{q} + \mathbf{Q}\cdot\begin{bmatrix} F_C / M \\ M_C / M \end{bmatrix}\right\} - \frac{1}{\phi_a}\begin{bmatrix} \lambda_{cyl0} \\ -\lambda_{cyl0} \end{bmatrix} \tag{15}$$

$\lambda_{cyl\,0}$ was earned by rewriting the compliance for a beam on an elastic foundation [6] into a form of a cylindrical shell.

$$\lambda_{cyl0} = \frac{1}{\beta D}\cdot\frac{\cosh \beta H - \cos \beta H}{\sinh \beta H + \sin \beta H} \tag{16}$$

3. NUMERICAL ILLUSTRATION

Some solutions are illustrated in this chapter to show the validity of the closed form equations of SIF derived above. At the same time, the characteristics of the SIF will be demonstrated.

3. 1 The SIF for a case in which the crack surface is subjected to M_0

A case shown in Figure 3 will be illustrated. The SIF, K_{M0}, calculated by Eq. (14) was compared with the value K_{FEM} by FEM. (M_C/M_0) and ϕ_a in Eq. (14) was calculated by Eq. (9) and (13), respectively.

The case that was investigated is as follows. The cylinder has mean radius R_m = 105 mm and thickness W = 10 mm for all cases, and, as to material constants, Young's Modulus E = 206 GPa, Poisson's Ratio ν = 0.3 are commonly used. Two cases were investigated for the total length of the cylinder, that is H = 40, 100 mm. For each H, crack location was varied as h_1/H = 0.5, 0.625 and 0.75. The calculated results are normalized by K_{M0beam} (Eq. (14)), which is the SIF of a single edge cracked strip under pure bending moment M_0, and are compared in Figure 4.

From this figure, the following can be deduced.

1. These two solutions show good agreement in a practical sense even when H/W = 4 and h_1/H = 0.75, therefore, $h_2 = W$. It can be expected in general, in agreement with the beam theory, that the longer the beam is, the more accurate the solution will be. However, it seems that the method can be applied even to the case where h_1 or h_2 becomes comparable to W and the beam theory no longer is valid.
2. The SIF in interest, K_{M0} becomes smaller for a longer cylinder. In this situation, there may be a risk in evaluating K_{M0} of a finite length cylinder by using the result for an infinite length.
3. K_{M0} becomes large when the crack is located near the edge of a cylinder.
4. It is necessary to consider the effects of the cylinder length and crack location appropriately, in evaluating the SIF of a circumferentially cracked cylinder under axisymmetric bending.

The correction factor for finite width F_M [5] and compliance $\Delta\lambda$ [7], which were used in the numerical example are as follows.

$$F_M(\xi)=\sqrt{\frac{2}{\pi\xi}\tan\frac{\pi\xi}{2}}\cdot\frac{0.923+0.199[1-\sin(\pi\xi/2)]^4}{\cos(\pi\xi/2)} \tag{17}$$

$$\Delta\lambda(\xi)=\frac{\pi(1.1215)^2}{2E}\cdot\frac{\xi^2}{(1-\xi)^2(1+2\xi)^2}\times[1+\xi(1-\xi)(0.44+0.25\xi)]\left(\frac{6}{W}\right)^2 \tag{18}$$

3. 2 Parametric study : Effects of the cylinder length and crack location on the SIF

(A case of an inner circumferential surface crack in a cylinder subjected to a pair of bending load on the edge)

In this section, the SIF K_M of the case shown in Figure 2 was examined. The effects of the cylinder length and the crack location on K_M were investigated by applying Eq. (10). (M_C/M_0) in Eq. (14) was calculated by Eq. (9). The cylinder configuration and the material constants in this investigation are identical to those in the previous clause.

Two cases corresponding to $a/W = 0.3$ and 0.6 are shown in Figure 5 and Figure 6, respectively. The calculated results K_M were normalized by K_{Mbeam} (Eq. (10)), which is the SIF of a single edge cracked strip under pure bending moment M. Note that K_M/K_{Mbeam} for small βH, or for small h_1/H, h_2/H are given for reference only, considering the fact that Eq. (10) was derived on the basis of the beam theory.

From these figures, the following can be deduced.

1. The values of K_M become nearly equal to those of K_{Mbeam}, when the cylinder becomes short, or when the crack location gets near the cylinder edge.
2. The closer the crack is located near the cylinder edge, the stronger the cylinder length affects the K_M.
3. The longer the cylinder is, the stronger the crack location affects the K_M.

4. CONCLUSIONS

In this paper, closed form equations of SIF and of inclination angle at the cylinder edge for an arbitrarily located circumferential crack in a cylinder, subjected to axisymmetric bending loads, were derived. Two cases, that is, the first case where bending loads of the same value are applied on the both ends of the cylinder, and the second case for a given crack face traction equal to the normal stress acting on the crack plane when the body is uncracked, were considered. The effects that the cylinder length and the crack location have on the SIF and inclination angle, can be evaluated by the equations (10), (14), (11), and (15).

The validity of the equation was illustrated by comparing the solutions with the numerical ones, for a problem under a traction equivalent to an axisymmetric bending moment on the cracked surface.

The results showed that the SIF increases as the cylinder length decreases, and as the crack gets near the cylinder edge. These results indicate the necessity to take into account the effects of cylinder length and crack location appropriately, in evaluating SIF of a inner circumferential surface crack in a cylinder under axisymmetric bending.

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APPENDIX 1

Deformation of a beam on an elastic foundation of length h under lateral loads P_1, P_2 and bending loads M_1, M_2 on its ends is considered (Figure A. 1) [4]. The loads are defined as values per unit thickness. Let the left and right ends of the beam be named points A and B, respectively. Here we focus our attention on point X, whose distance to points A and B are x and x', respectively ($x + x' = h$). The deflection y and inclination angle θ at the point X are expressed by the following equation.

$$\begin{bmatrix} y(x,x') \\ \theta(x,x') \end{bmatrix} = \Lambda(x,x') \cdot \begin{bmatrix} P_1 \\ M_1 \end{bmatrix} + \Lambda_*(x,x') \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \quad \textbf{(A 1)}$$

The matrixes $\Lambda(x, x')$ and $\Lambda_*(x, x')$ are defined in the following Eqs. (A 2) and (A 3).

$$\Lambda(x,x') = \begin{bmatrix} \lambda_{yP}(x,x') & \lambda_{yM}(x,x') \\ \lambda_{\theta P}(x,x') & \lambda_{\theta M}(x,x') \end{bmatrix} \quad \textbf{(A 2)}$$

$$\Lambda_*(x,x') = \begin{bmatrix} \lambda^*_{yP}(x,x') & \lambda^*_{yM}(x,x') \\ \lambda^*_{\theta P}(x,x') & \lambda^*_{\theta M}(x,x') \end{bmatrix} = \begin{bmatrix} \lambda_{yP}(x',x) & \lambda_{yM}(x',x) \\ -\lambda_{\theta P}(x',x) & -\lambda_{\theta M}(x',x) \end{bmatrix} \quad \textbf{(A 3)}$$

where λ_{yP}, for instance, denotes the compliance that relates the deflection y at point X caused by P_1 to P_1, and λ^*_{yP} denotes the compliance that relates the deflection y at point X caused by P_2 to P_2. The other compliances are defined in a similar way.

Each component of the matrixes are given by Eqs. (A 4) ~ (A 7) concretely, by regarding the beam in Figure A. 1 as the beam equivalent to a cylinder in Figure 1(b).

$$\lambda_{yP}(x,x') = \frac{1}{2\beta^3 D} \times \frac{\sinh\beta h \cos\beta x \cosh\beta x' - \sin\beta h \cosh\beta x \cos\beta x'}{\sinh^2\beta h - \sin^2\beta h} \quad \textbf{(A 4)}$$

$$\lambda_{\theta P}(x,x') = -\frac{1}{2\beta^2 D} \times \frac{1}{\sinh^2\beta h - \sin^2\beta h} \times [\sinh\beta h(\sin\beta x \cosh\beta x' + \cos\beta x \sinh\beta x') + \sin\beta h(\sinh\beta x \cos\beta x' + \cosh\beta x \sin\beta x')] \quad \textbf{(A 5)}$$

$$\lambda_{yM}(x,x') = \frac{1}{2\beta^2 D} \times \frac{1}{\sinh^2\beta h - \sin^2\beta h} \times [\sinh\beta h(\sin\beta x \cosh\beta x' - \cos\beta x \sinh\beta x') + \sin\beta h(\sinh\beta x \cos\beta x' - \cosh\beta x \sin\beta x')] \quad \textbf{(A 6)}$$

$$\lambda_{\theta M}(x,x') = \frac{1}{\beta D} \times \frac{\sinh\beta h \cos\beta x \cosh\beta x' + \sin\beta h \cosh\beta x \cos\beta x'}{\sinh^2\beta h - \sin^2\beta h} \quad \textbf{(A 7)}$$

The compliances in Eqs. (5) and (6) were given by applying these Eqs. from (A 4) to (A 7), considering an additional information $x + x' = h$. For example, $\lambda_{yP}(0, h_2)$ is earned by substituting $h = h_2$, $x = 0$ and $x' = h_2$ in Eq. (A 4). $\lambda^*_{yP}(h_1, 0)$ is earned by applying the relation described in Eq. (A 3), that is $\lambda^*_{yP}(h_1, 0) = \lambda_{yP}(0, h_1)$, and by substituting $h = h_1$, $x = 0$ and $x' = h_1$ in Eq. (A 4).

APPENDIX 2 Nomenclature

a: crack length (m)

W: cylinder thickness (m)

ξ: $= a/W$

H, h: cylinder length (m)

R: radius (m)

E: Young's Modulus (MPa)

ν: Poisson's Ratio

β: characteristic of the system (1/m)

D: equivalent flexural rigidity of cylinder (N・m^2/m)

K: stress intensity factor (MN/m$^{3/2}$)

P: axisymmetric load per unit circumferential length (MN/m)

F: axisymmetric force per unit circumferential length (MN/m)

F_M: correction factor of finite width for single edge cracked strip under pure bending

M: axisymmetric bending load or moment per unit circumferential length (MN・m/m)

y: deflection (m)

θ: inclination angle (rad)

λ_{yP}, λ^*_{yP}: compliance that relates deflection y caused by load P (m/(MN/m))

λ_{yM}, λ^*_{yM}: compliance that relates deflection y caused by bending load M (m/(MN・m/m))

$\lambda_{\theta P}$, $\lambda^*_{\theta P}$: compliance that relates inclination angle θ caused by load P (rad/(MN/m))

$\lambda_{\theta M}$, $\lambda^*_{\theta M}$: compliance that relates inclination angle θ caused by bending load M (rad/(MN・m/m))

LIST OF FIGURES

Figure 1 Replacement of axisymmetric deformation problem of a cylinder by that of two beams on an elastic foundation connected with a spring

Figure 2 A circumferentially cracked cylinder subjected to axisymmetric bending load pair

Figure 3 A cylinder with a circumferentially cracked surface subjected to M_0

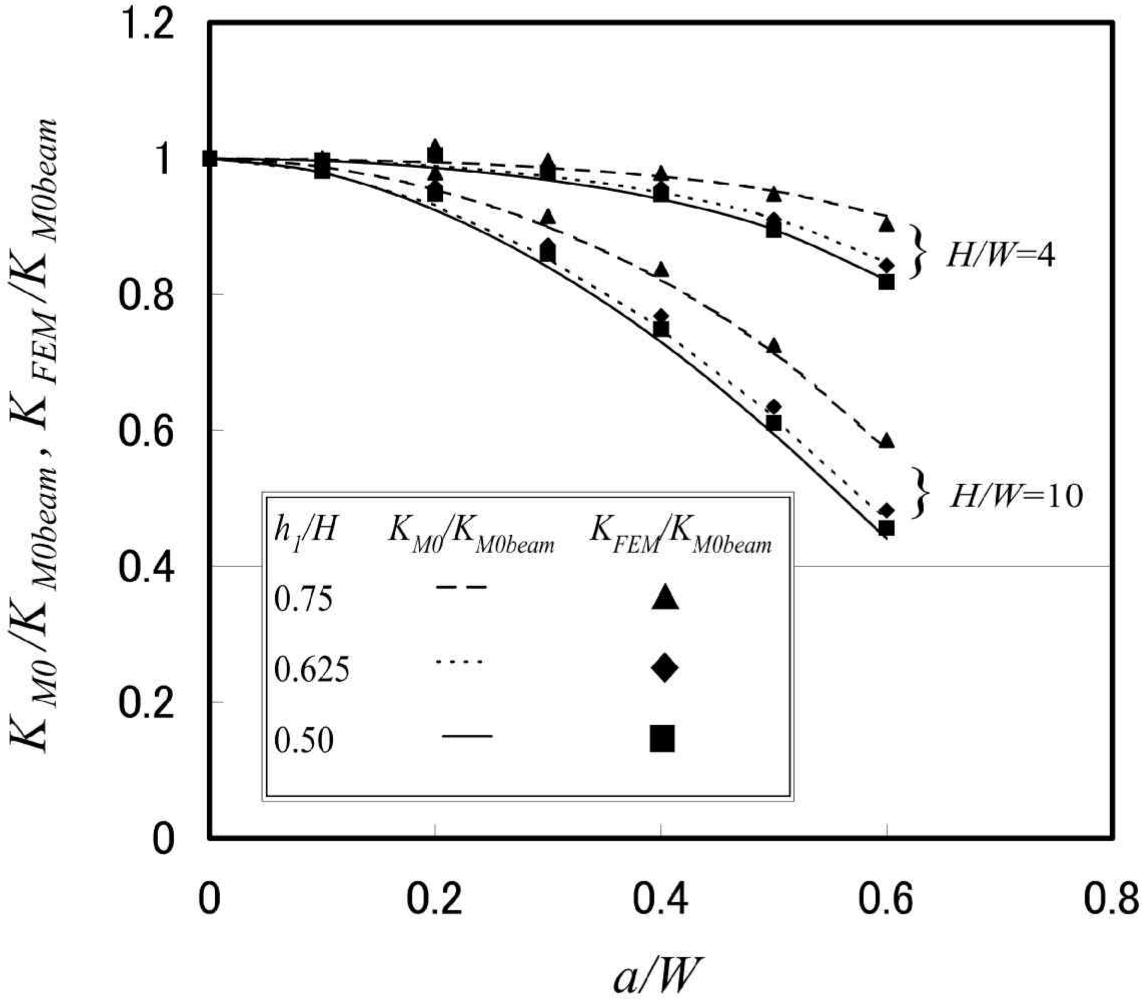

Figure 4 Comparison of the SIF from the closed form equation with that from FEM

Figure 5 The effects of cylinder length and crack location on the SIF (a/W = 0.3; R_m/W = 10.5, W = 10 mm)

Figure 6 The effects of cylinder length and crack location on the SIF (a/W = 0.6; R_m/W = 10.5, W = 10 mm)

Figure A. 1 A beam on an elastic foundation loaded on its ends