Textures and NMR Spectra of a Superfluid 3He-A Phase Confined in Rotating Narrow Cylinders

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Abstract

We confine a superfluid ^{3}He-A phase in narrow cylinders, these radii are 0.05mm and 0.1mm. The system is rotated around the cylinder axis, and its rotation velocity ranges between -6.28 rad/sec and 6.28rad/sec. The strong magnetic field (22mT) is applied along cylinder axis. In these systems, we determined $\boldsymbol{l}$- and $\boldsymbol{d}$- textures by minimizing GL-free energies. In the case of the small cylinder (R=0.05mm) a dipole locked and a dipole unlocked Mermin-Ho textures are obtained as local minimum free energy states. In the case of the large cylinder (R=0.1mm) a single vortex and a three vortices states are obtained. The stability of each state depends on the rotating velocity. A vortex charge change of two quanta occurs by a pair of continuous vortices coming in and going out. For each case, the NMR spectrum of the transverse resonance mode is calculated by using a finite element method (FEM). Using the FEM the boundary condition of spin wave mode is strictly imposed and the higher resonance mode is also calculated. The calculated result well agrees with empirical data.

Key words: superfluidity, helium3, texture, vortex, NMR

1. Introduction

Superfluid ^{3}He has a spin triplet P-wave Cooper pair. It shows various fascinating physical phenomena due to the internal degree of freedom in Cooper pairs. Especially, the Cooper pair in the A-phase has an intrinsic angular momentum $\boldsymbol{l}$ in the orbital space. Spatial variation of $\boldsymbol{l}$-vector is coupled with the Cooper pair phase (gauge-orbital space coupling) and it makes a superflow. Hence we have to consider the quantization condition and a spatial change of the $\boldsymbol{l}$-vector simultaneously in the A-phase vortex. Various continuous quantum vortices appear in the A-phase because of this gauge-orbital space coupling effect.[1][2] The appearing vortex is determined by the magnetic field strength and rotation velocity. In the bulk system the modified Anderson-Toulouse textures are the unit structure of the continuous vortex which possesses a vortex charge $p = 2$ On the other hand another possible continuous texture, Mermin-Ho texture, has a vortex charge $p = 1$. But this texture has not been observed in bulk system. Therefore we plan to realize and observe continuous vortices which have odd vorticity. On this purpose, we prepare the rotating narrow cylinder system. In this geometry the odd vorticity vortex is expected to be stabilized due to the boundary condition. We prepare the trial textures which are the candidates of the stable structure. Then we determine the optimum textures which minimizes free energy by utilizing local down hill method. Next we determine the transverse NMR spectra with using finite element method(FEM) and compare the obtained results with the empirical data.

2. Model

We confine the superfluid ^{3}He A-phase in the narrow cylinders. The radius R of the small cylinder is 0.05mm and big one is 0.1mm. The characteristic rotation velocity $\omega_{\rm c}$ of the cylinder is given in $\omega_{\rm c} = \hbar/(2m_3R^2)$, where m_3 stands for ^{3}He particle mass. Each characteristic velocity of cylinders is 1.06rad/sec for $R = 0.1$mm and 4.24rad/sec for $R = 0.05$mm. We assume that the wall of the cylinder has no magnetic character. Therefore the wall has no effect on determining the orientation of $\boldsymbol{d}$-vectors on the wall. On the other hand the $\boldsymbol{l}$-vector is directed to parallel to the surface normal on the wall. Configuring the coordinate system, we adopt the z-axis parallel to the cylinder axis. We rotate the cylinder along the cylinder axis with angular velocity $\omega \boldsymbol{e}_z$. In this situation, the normal fluid of the system is rotated with a container. Then its velocity becomes $\boldsymbol{v}_{\rm n} = \omega \boldsymbol{e}_z \times \boldsymbol{r}$. We applied a magnetic field $\boldsymbol{H}$ along cylinder axis, $\boldsymbol{H} = H\boldsymbol{e}_z$. The intensity of the magnetic field is 22mT, and it is strong enough to overcome the dipole free energy.

The order parameter of the superfluid ^{3}He is specified by the spin space vecotr $\boldsymbol{d}$ and the orbital space triad vectors $\{\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}\}$. The 3×3 representation of the order parameter $A_{\mu j}$ is given with using $\boldsymbol{d}, \boldsymbol{m}$ and $\boldsymbol{n}-$vectors,

$$A_{\mu j} = \Delta\, d_\mu\,(m_j + {\rm i} n_j) \tag{1}$$

The superfluid velocity $\boldsymbol{v}_{\rm s}$ is discribed by phase gradient of the order parameter as

$$\boldsymbol{v}_{\rm s} = \frac{\hbar}{2m_3} m_j \nabla\, n_j, \tag{2}$$

where repeated indices mean summation of x, y, z-components. The total free energy density $f_{\rm tot}$ of the system is ginven:

$$f_{\rm tot} = f_{\rm dip} + f_{\rm mag} + f_{\rm kin}, \tag{3}$$

where the terms $f_{\rm dip}, f_{\rm mag}, f_{\rm kin}$ stand for dipole, magnetization and kinetic free energies, respectively. Each free enegy term is given:

$$f_{\rm dip} = -g_{\rm d}\, \Delta^2 (\boldsymbol{d}\cdot\boldsymbol{l})^2, \tag{4}$$

$$f_{\rm mag} = g_{\rm h}\, \Delta^2 (\boldsymbol{d}\cdot\boldsymbol{H})^2, \tag{5}$$

$$\begin{aligned} 2f_{\rm kin} = {} & \rho_\perp \boldsymbol{v}^2 - (\rho_\perp - \rho_\parallel)(\boldsymbol{l}\cdot\boldsymbol{v})^2 + 2C\,\boldsymbol{v}\cdot(\nabla\times\boldsymbol{l}) \\ & -2C_0\,(\boldsymbol{v}\cdot\boldsymbol{l})\,(\boldsymbol{l}\cdot\nabla\times\boldsymbol{l}) + K_{\rm s}(\nabla\cdot\boldsymbol{l})^2 \\ & +K_{\rm t}(\boldsymbol{l}\cdot\nabla\times\boldsymbol{l})^2 + K_{\rm b}\mid \boldsymbol{l}\times(\nabla\times\boldsymbol{l})\mid^2 \\ & +K_5 \mid (\boldsymbol{l}\cdot\nabla)\,\boldsymbol{d}\mid^2 \\ & +K_6 \sum_{i,j} \left[(\boldsymbol{l}\times\nabla)_i d_j\right]^2. \end{aligned} \tag{6}$$

The kinetic free energy term is so called Cross term.[4] We use the Cross coefficients calculator which is served by R. Hänninen.[5] We can take into account temperature and pressure dependence of the Cross coefficients by thanks of this calculator. Each amplitude of the coefficient is mentioned in the ref.[6] In the kinetic energy term the notation $\boldsymbol{v}$ means the difference between the superflow velocity and that of the normal fluid : $\boldsymbol{v} = \boldsymbol{v}_{\rm s} - \boldsymbol{v}_{\rm n}$.

The numerical calculation is carried out to minimize tota free energy $F_{\rm tot}$,

$$F_{\rm tot} = \int {\rm d}^2 r\, f_{\rm tot}[\,\boldsymbol{d}(\boldsymbol{r}), \boldsymbol{m}(\boldsymbol{r}), \boldsymbol{n}(\boldsymbol{r})\,]. \tag{7}$$

In order to solve this variational problem, we use a local down hill method.[7] To get the optimum texture we prepare several initial configurations of order parameters, and lower the free energy of each configurations. Then we compare each minimized free energies and adopt the lowest one.

3. NMR Spectrum

After determining the optimum texture, we calculate a spin wave mode transverse NMR spectra by using the FEM. In order to calculate transverse spin wave NMR spectrum, we apply main magnetic field along z-axis. The rf-field is applied in the cross section of the cylinder.

We denote the x, y-coordinate dependent amplitude of spin wave vibration as $g(x, y)$. Two terms contain the second order of the free energy density variation of $g(x,y)$: one is the kinetic term $\delta^2 f_{\rm kin}$ and the other is the potential term $\delta^2 f_{\rm pot}$.[3]

$$\delta^2 f_{\rm kin} = K_6(g_{,x}^2 + g_{,y}^2) \\ +(K_5 - K_6)(l_x^2 g_{,x}^2 + l_y^2 g_{,y}^2 + 2\, l_x l_y g_{,x} g_{,y}) \quad (8)$$

$$\delta^2 f_{\rm pot} = \Bigg[(\boldsymbol{l}\cdot\boldsymbol{d}_0)^2 - (\boldsymbol{l}\cdot\boldsymbol{e}_\theta)^2 + \xi_D^2 \Big\{ K_5 \mid (\boldsymbol{l}\cdot\nabla)\boldsymbol{d}\mid^2 + K_6 \sum_{i,j}\big[(\boldsymbol{l}\times\nabla)_i d_j\big]^2\Big\}\Bigg] g^2. \quad (9)$$

K_5, K_6 are the coefficients of the Cross term. Vector $\boldsymbol{d}_0$ stands for an equilibrium direction of $\boldsymbol{d}-$vector. Unit vector $\boldsymbol{e}_\theta$ is directed to the polar angle direction which means the vibration direction of $\boldsymbol{d}-$vector. The parameter ξ_D is the dipole cohearent length ($\sim 10\mu$m) It should be noted that we neglect a dissipation mechanism in this formulation. The Lagrangean of the spin wave mode in the cylinder is given

$$L[g(x,y)] = \int {\rm d}^2 r \Big\{\delta^2 f_{\rm kin}[g(x,y)] + \delta^2 f_{\rm pot}[g(x,y)]\Big\}. \quad (10)$$

The eigen values and eigen spin wave functions are obtained by solving the variational equation,

$$\delta L[g(x,y)] = 0. \quad (11)$$

The equation have to be solved under the constraints:

$$\int {\rm d}^2 r\, g^2(x,y) = 1, \quad (12)$$

$$\big[\,\boldsymbol{n}_{\rm wall}\cdot\nabla g(x,y)\,\big]_{r=R} = 0, \quad (13)$$

where $\boldsymbol{n}_{\rm wall}$ means a surface normal vector on the wall. This constraint means that the spin current does not exist on the wall and gives a quantization condition against spin wave modes.

This variational problem is solved by utilizing a standard FEM procedure. The boundary condition (13) is satisfied spontaneously in using FEM because this condition is a natural/default condition of FEM. This point is the largest merit in using FEM. A generalized eigen equation of a matrix form is solved in the FEM of the spin wave function. Then the lowest mode and all the higher mode of the resonance frequencies and wave functions are obtained.

4. Calculation Result and Discussion

We carry out the numerical calculation in the cases of $R = 0.05$mm and $R = 0.1$mm cylinders in the rotation velocity range -6.28rad/sec$< \omega <$ 6.28rad/sec.

4.1. *In the case of the $R = 0.05\mu m$ cylinder*

We calculate textures and NMR with using the parameters: $P = 31.5$bar, $T = 0.75T_{\rm c}$. In all the angular velocity range, the dipole unlocked Mermin-Ho texture is stable in this cylinder.[8] The obtained texture is shown in fig.1 and fig.2. This configuration has hyperbolic shape of $\boldsymbol{d}$-texture lying in xy-plane and a dipole unlocked region exists considerably. Thus this texture has the disadvantage in dipole free energy but has an advantage in the $\boldsymbol{d}-$field gradient energy. Under the condition $\omega = 0$, the calculated lowest spin wave resonance frequency shift $R_{\rm t}^2$ is $R_{\rm t}^2 = 0.32$. It is well agreed with experimental results.[9] Also metastable textures ware found in the experiments. This metastable texture should be an axial symmetric Mermin-Ho texture which has the axial symmetric $\boldsymbol{d}-$fields. In the calculation, this texture becomes metastable against dipole unlocked texture. Its calculated spin wave frequency shift is $R_{\rm t}^2 = 0.59$ and also well agreed with the empirical data.

4.2. *In the case of the $R = 0.1\mu m$ cylinder*

Calculation is done under the condition $P = 31.5$bar, $T = 0.82T_{\rm c}$. Under the condition $\omega = 0$, axial symmetric Mermin-Ho texture becomes stable. But the free energy difference between this texture and that of the dipole locked texture is small. Therefore more precise calculation should be required especially in the large field gradient regions in order to determine a ground state configuration.

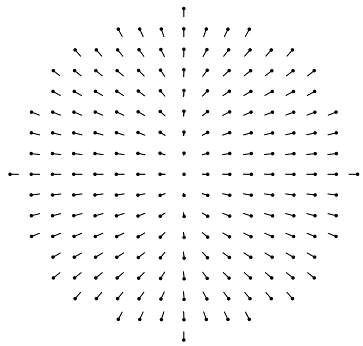

Fig. 1. The $\boldsymbol{l}-$field of dipole unlocked Mermin-Ho texture.

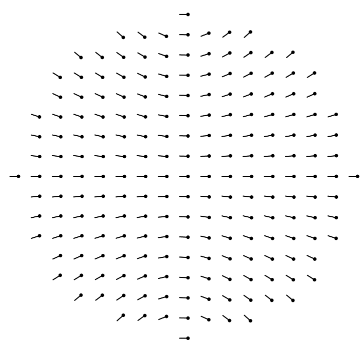

Fig. 2. The $\boldsymbol{d}-$fields of dipole unlocked Mermin-Ho texture.

The calculated NMR shifts are $R_t^2 = 0.75$ for the axial symmetric Mermin-Ho texture and $R_t^2 = 0.39$ for the dipole unlocked Mermin-Ho texture. Nevertheless these spin wave satellite peaks were not observed experimentally. The scale of spin wave resonance region is ξ_D. Therefore the resonance region of spin wave against the cross section becomes very small in the case of $R = 0.1$mm cylinder. It is the reason of the observation difficulty.

In the rotation velocity region 3.5rad/sec$< \omega$, 3-vortices states become stable. This 3-vortices state have a degree of freedom of each vortex position in the cylinder. Then many metastable configurations appear, and it is difficult to determine the lowest free energy configuration. One of the 3 vortices configuration is shown in fig.3. The calculated lowest NMR shift is $R_t^2 = 0.40$ and that of the ex-

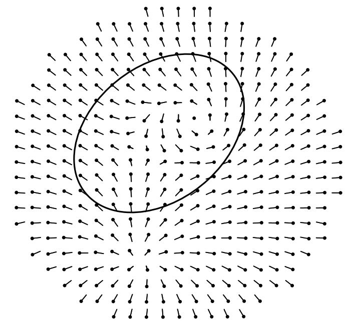

Fig. 3. The $\boldsymbol{l}-$field of 3-vortices configuration under the rotation velocity $\omega = 5.3$rad/sec. A CUV vortex exists in a ellipse.

perimental data is $R_t^2 = 0.31$. More precise vortices formations should be considered.

Acknowledgements

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