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ABSTRACT

A weight function to evaluate the stress intensity factor (SIF) of a circumferential crack, subjected to arbitrarily distributed stress on the crack surfaces, in a finite length thin-walled cylinder was derived based on the closed form SIF equation previously developed by the authors. It is easy to evaluate the effects of structural parameters and stress distribution on the SIF with this weight function. Numerical examples confirmed the validity of the weight function. These examples showed that the effect of cylinder length on the SIF is quite large.

KEY WORDS: Fracture Mechanics, Stress Intensity Factor, Weight Function, Thin-Walled Cylinder Shell, Circumferential Crack and Finite Length Cylinder.

1. INTRODUCTION

The stress intensity factor (SIF) of a circumferential crack in a cylinder is one of the fundamental quantities to evaluate the reliability of cracked pressure vessels and cracked pipes, etc. Among the various loading conditions to be considered in calculating the Mode I SIF for the crack, arbitrarily distributed normal stress on the crack surfaces is one of the most fundamental and important ones. For example, the typical thermal stress problem of this configuration, in which we assume radial temperature distribution, can be treated as a superposition of several problems under external loads, including a problem of an arbitrary normal stress distribution on the cracked surfaces (Meshii and Watanabe, 1998a). There are various numerical methods, such as finite element method (FEM), suitable to calculate the SIF of the problem. But there is a drawback to most of them when we want to evaluate the effect of the stress distribution or structural parameters on the SIF systematically, concerning time and cost. We thought that the weight function method is an effective means to easily overcome the drawback above as long as the weight function of the problem is given.

The weight function method, which Bueckner (1970) and Rice (1972) developed independently, is a useful method to obtain the SIF of a cracked body. Once the weight function is derived for a set of cracked body configuration and loading boundary, the SIF of the arbitrarily loaded body on an identical boundary can be calculated by boundary integration. The weight function for the configuration itself is derived from the SIF under a certain loading condition and the corresponding derivative of the displacement on the loading boundary with respect to the crack length. Though the procedure to derive the weight function is clear, the necessary displacement solution for a crack problem is not always easy to obtain. This difficulty seems to have prevented this method from becoming a leading method in SIF calculations.

As to a circumferential crack in a cylinder, Labbens et al. (1976) have derived the weight function of the crack for a long and thin cylinder through FEM, and Cheng and Finnie (1985) for short and thin cylinder through compliance method. Nied and Erdogan (1983) made an analytical approach by assuming an infinitely long cylinder. But, as Labbens et al. (1976) pointed out, the effect of the cylinder length on the SIF should be properly considered for short cylinders with length $H < 5/\beta$ (β is a quantity which is used in replacing cylindrical shell by a beam on an elastic foundation, and its definition is shown in the APPENDIX). Cheng and Finnie's weight fuction (1985) will be a candidate for a very short cylinder of $H/W \leq 1.5$, where W is the wall thickness. However, since long or short cylinder assumption is in fact often not satisfied, we thought a weight function considering the cylinder length is necessary.

In this paper, we derived the weight function of a circumferential crack in a thin-walled cylinder with arbitrarily distributed stress on the crack surface. It is easy to evaluate the effect of the structural parameters on the SIF, such as cylinder length, with this weight function. Then we evaluated the SIFs of the configuration under fundamental loading types and demonstrated the effects of structural parameters on the SIF. In concrete, we first introduced our closed form SIF equation (1998b) for a circumferential crack in a finite length thin-walled cylinder with linear stress distribution (axisymmetric bending) on the crack surfaces. Subsequently, we obtained the corresponding approximate crack surface displacement solution for the problem by applying the Petroski and Achenbach method (1978). We then derived the desired weight function by using the displacement and our closed form SIF equation. Finally, we examined the validity of the weight function by comparing the SIFs of the crack under specific loading conditions obtained by applying the derived weight function with the corresponding solution obtained through other methods.

2. SIF FOR LINEAR STRESS DISTRIBUTION AND DISPLACEMENT ON THE CRACK SURFACE

The final goal of this paper is to evaluate the SIF of the problem in Fig. 1. A circumferential crack in a thin-walled cylinder, whose cylinder length is H, is subjected to an arbitrary normal stress distribution $\sigma(x)$ on the crack surfaces. In case $\sigma(x)$ is linear and crack location is arbitrary in the axial direction, we have a closed SIF equation as shown in the APPENDIX (Meshii and Watanabe, 1998b). This SIF solution was derived through a method similar to the compliance method which Cheng and Finnie used in deriving their weight function (1985), except on the point that we handled the problem as statically indeterminate and overcame the restriction of $H/W \leq 1.5$ which Cheng and Finnie's solution had. Let us consider a special case for our SIF solution as shown in Fig. 2. By setting $h_1 = h_2 = H/2$ in our previous SIF equation (Meshii and Watanabe, 1998b) and introducing the structural parameter $\psi_f(\xi, \beta H)$ defined in the APPENDIX, the SIF for this case is obtained as follows.

$$K_{M0}(a) = \psi_f(\xi, \beta H) \cdot \left\{ \frac{M_0}{Z} \sqrt{\pi a} \cdot F_M(\xi) \right\} \tag{1}$$

Here, $\xi = a/W$ is non-dimensional crack length, $Z = W^2/6$ the section modulus and F_M the infinite length edge-cracked strip's correction factor for finite width under pure bending. M_0 is a moment related to the linear stress distribution $\sigma(x)$, whose inner and outer stress difference is $2\sigma_0$, by the following equations, which include the curvature effect.

$$\sigma(x)=\sigma_0 \cdot \left[1+W/(6R_m)-2x/W\right] \tag{2}$$

$$\frac{M_0}{Z}=\sigma_0 \cdot \left[1-\frac{1}{12}\left(\frac{W}{R_m}\right)^2\right] \Big/ \left[1+\frac{1}{12}\left(\frac{W}{R_m}\right)^2\right] \tag{3}$$

The closed form equation (1) for the case in Fig. 2 shows that we can evaluate the SIF for the problem as the SIF of an edge cracked beam under pure bending $\psi_f(\xi, \beta H)\cdot M_0$ and, at the same time, we can approximate the stress distribution around the crack for this problem by the stress distribution of an edge cracked strip under pure bending $\psi_f(\xi , \beta H)\cdot M_0$. This means also that one can approximate the deformation near the crack in Fig. 2 in the same manner. Incidentally, we know that Tada et al. (1985) gave the opening displacement at the crack end, δ_{beam}, of an edge-cracked strip under pure bending in a closed form equation concretely given by Eq. (10) in the next chapter. So, by utilizing this δ_{beam}, we derived a closed form equation that gives the approximate displacement on the crack surface for the problem in Fig. 2.

First, we assumed that the opening displacement at the crack end $\delta(a)$ for the problem in Fig. 2 is given by

$$\delta(a)=\psi_f(\xi, \beta H)\cdot \delta_{beam}(a) \tag{4}$$

Next, we applied the method first proposed by Petroski and Achenbach (1978), and assumed that we can approximate the displacement in the y direction u_y by a function of the polar coordinate (r, θ) of which the origin is the crack tip point, as follows,

$$u_y=\sum_{n=1}^{\infty}\frac{A_n}{2G}r^{n/2}\times\left\{\kappa\sin\frac{n\theta}{2}+\frac{n}{2}\sin\left(\frac{n}{2}-2\right)\theta-\left[\frac{n}{2}+(-1)^n\right]\sin\frac{n\theta}{2}\right\} \tag{5}$$

Here G is the modulus of elasticity in shear, κ is a constant which represents $(3-\nu)/(1+\nu)$ for the plane stress and

(3-4ν) for plane strain, where ν is Poisson's ratio. When we sum n in Eq. (5) up to n = 3, we are able to deduce u_y on the crack surface as follows, considering on the crack surfaces to be $\theta = \pm\, \pi$.

$$u_y = \pm\frac{\kappa+1}{2G}\sqrt{r}\left(A_1 - A_3 r\right) \tag{6}$$

The ± sign in Eq. (6) corresponds to upper and lower surfaces of the crack, respectively. Since $A_1/\sqrt{2\pi}$ is the SIF $K_{M0}(a)$, considering $r = a - x$, we deduced the crack opening displacement (COD) along the crack surface υ, when $C_3 = A_3/\sqrt{2\pi}$, as follows.

$$\upsilon(x;a) = \frac{\kappa+1}{G}\sqrt{\frac{a-x}{2\pi}}\cdot\left[K_{M0}(a) - C_3\cdot(a-x)\right] \tag{7}$$

By considering the fact that υ at x = 0 is equal to the opening displacement at the crack end $\delta(a)$, we obtained the coefficient C_3 from Eq. (7) as follows.

$$C_3 = \frac{1}{a}\left(K_{M0}(a) - \frac{G}{\kappa+1}\sqrt{\frac{2\pi}{a}}\cdot\delta(a)\right) \tag{8}$$

We obtained the desired COD along the crack surface υ by substituting Eq. (8) into Eq. (7).

$$\upsilon(x;a) = \frac{\kappa+1}{G}\sqrt{\frac{a-x}{2\pi}}\cdot\left[K_{M0}(a) - \frac{a-x}{a}\left(K_{M0}(a) - \frac{G}{\kappa+1}\sqrt{\frac{2\pi}{a}}\cdot\delta(a)\right)\right] \tag{9}$$

3. DERIVATION OF THE WEIGHT FUNCTION

In this section, we first examined the validity of the closed form COD (Eq. (9)) of a circumferential crack in a thin-walled cylinder subjected to linear stress distribution, and then used it to derive the desired weight function .

3. 1 COD of circumferential crack in a thin-walled cylinder subjected to linear stress distribution

As the first step, we examined the validity of Eq. (4), which is the closed form equation for the COD at the crack end δ of the circumferential crack in a thin-walled cylinder subjected to a linear stress distribution of $2\sigma_0$ = 19.6 MPa in Eq. (2), by comparing the δ with that obtained by FEM. The equivalent moment M_0 for this case is calculated by Eq. (3).

The geometrical dimensions and material constants of the cylinder used for this study are as follows: R_m = 105 mm, W = 10 mm, H = 40, 100 mm, and G = 79.2 GPa, ν = 0.3. For each H, we took crack lengths of a = 1, 2, 3, 4, 5 and 6 mm. The results δ by Eq. (4) are normalized by δ_{beam} (the crack opening displacement at the crack end for edge-cracked strip under pure bending M_0) and shown in Fig. 3. The δ_{beam} (Tada, et al. 1985) which we used in this normalization and $\Delta\lambda$ (Takahashi, 1991) which we used in the calculation of $\psi_f(\xi, \beta H)$ are as follows.

$$\delta_{beam}(a) = \frac{(\kappa+1)}{2G} \cdot \frac{M_0}{Z} \cdot a \cdot V(\xi) \qquad (10)$$

$$\text{where} \quad V(\xi) = 0.8 - 1.7\xi + 2.4\xi^2 + 0.66/(1-\xi)^2 \qquad (11)$$

$$\Delta\lambda(\xi) = \frac{\pi(\kappa+1)(1.1215)^2}{16G} \cdot \frac{\xi^2}{(1-\xi)^2(1+2\xi)^2} \times \left[1 + \xi(1-\xi)(0.44+0.25\xi)\right]\left(\frac{6}{W}\right)^2 \qquad (12)$$

We conclude from Fig. 3 that the two solutions show good agreement in a practical sense.

Subsequently, we compared the COD along the crack surface υ_{FEM} from FEM (for the case of H/W = 4, a/W = 0.6) with υ by Eq. (9) in Fig. 4. The dotted line υ_K in the figure represents υ corresponding to the $r^{1/2}$ term in Eq. (9), which we showed for reference.

We conclude from Fig. 4 that the approximated COD along the crack surface subjected to a linear stress distribution, υ (given by Eq. (9)), shows good practical agreement with that obtained by FEM, υ_{FEM}. Note that the validity of the approximated υ is to be finally verified by comparing the SIF calculated by the weight function obtained from this υ with the SIF obtained by other methods.

3. 2 Derivation of the weight function

Once we obtain the COD along the crack surface under a certain loading condition, we are able to derive the desired weight function from the corresponding SIF and the derivative of this COD with respect to crack length a. And once we have successfully derived the weight function for a set of cracked body configurations and a loading boundary, we are able to evaluate the SIF of the arbitrarily loaded cracked body on the identical boundary by boundary integration.

Let us consider a case such as in Fig. 1, where load and structure are symmetric about the loading boundary, that is, the crack surface. Here we found it convenient to define the desired weight function in terms of the crack opening displacement along the crack surface υ. Considering the fact that υ is two times the absolute value of the displacement u_y on the crack surface, we are able to write the weight function $w(x; a)$ and the SIF for an arbitrary stress distribution $\sigma(x)$ on the crack surface as follows (Bueckner, 1970; Rice, 1972).

$$w(x;a) \equiv \frac{4G}{(\kappa+1)K_{M0}(a)} \cdot \frac{\partial \upsilon(x;a)}{\partial a} \qquad (13)$$

$$K(a) = \int_0^a \frac{R_i + x}{R_i + a} \sigma(x) \cdot w(x;a) dx \qquad (14)$$

Here, R_i is the inner radius of the cylinder.

By substituting Eqs. (1), (4) and (10) into Eq. (9) and applying the result to Eq. (13), we derived the desired weight function w as follows.

$$\begin{aligned} & w(x;a)\cdot[\sqrt{\pi}a^2W\sqrt{a-x}\cdot\psi_f\cdot F_M] \\ & =\sqrt{2}x\cdot F_M\cdot[Wx\cdot\psi_f+2a(a-x)\cdot\frac{\partial\psi_f}{\partial\xi}] \\ & +(a-x)\times\{2a\cdot\psi_f\cdot(\sqrt{2}x\cdot\frac{\partial F_M}{\partial\xi}+(a-x)\cdot\frac{\partial V}{\partial\xi})+V\cdot[W(2a+x)\cdot\psi_f+2a(a-x)\cdot\frac{\partial\psi_f}{\partial\xi}]\} \end{aligned} \tag{15}$$

4. NUMERICAL ILLUSTRATION

In this section, we confirmed the validity of the derived weight function (Eq. (15), for the circumferential crack in a finite length cylinder subjected to $\sigma(x)$ on the crack surfaces) by comparing the SIF which we calculated with Eq. (15) with the SIF which we evaluated by other methods such as FEM. The $\sigma(x)$ distributions which we considered in this section are linear, uniform and quadratic stress distributions. The cracked cylinder configuration and the material we used in this section are identical with those in section 3. 1.

4. 1 SIF under linear stress distribution

Here we consider the SIF of the circumferential crack in a thin-walled finite length cylinder subjected to linear stress distribution on the crack surface given for $2\sigma_0$ = 19.6 MPa in Eq. (2). We evaluated the SIF K_{weight} by applying the weight function given by Eq. (15) and compared it with $K_{M0}(a)$ obtained from Eq. (1). We chose $K_{M0}(a)$ as a reference value, because we thought if the COD given as Eq. (9) is accurate enough, K_{weight} should be very close to $K_{M0}(a)$. The calculated results are normalized by $K_{M0\ beam}$, which is the SIF for an edge-cracked strip under pure bending M_0, shown in Fig. 5. The $K_{M0\ beam}$ used for normalization is as follows (Tada et al, 1985).

$$K_{M0beam}(a)=\frac{M_0}{Z}\sqrt{\pi a}\cdot F_M(\xi) \tag{16}$$

$$F_M(\xi)=\sqrt{\frac{2}{\pi\xi}\tan\frac{\pi\xi}{2}}\cdot\frac{0.923+0.199\left[1-\sin(\pi\xi/2)\right]^4}{\cos(\pi\xi/2)} \tag{17}$$

Considering the fact the difference between K_{weight} and K_{M0} depends on the accuracy of the weight function and K_{M0} itself, we see that the difference between these SIFs is acceptable (maximum difference 3.2%). As a result, we conclude that the weight function given by Eq. (15) enables us to evaluate the SIF of the circumferential crack in a cylinder under linear stress distribution on the crack surface with sufficient practical accuracy. In addition, we see that the SIF is strongly affected by the cylinder length and that our weight function can properly handle this effect. The existence of this effect does not contradict with the guideline for long cylinder assumption $H/W \geq 5/(\beta W)$ (= 12.6), which Labbens et al. (1976) gave.

4. 2 Uniform stress distribution

Next we dealt with the SIF for the case in which the surface of the circumferential crack in a thin-walled cylinder is subjected to uniform stress distribution $\sigma(x) = \sigma_N = 9.8$ MPa. We compared the SIF K_{weight}, which we evaluated by Eq. (15), with the SIF K_{FEM}, which we evaluated from the FEM result. We show the calculated results normalized by $K_{N\ beam}$, which is the SIF for an edge-cracked strip under uniform stress σ_N, in Fig. 6. Here, the $K_{N\,beam}$ is

$$K_{Nbeam}(a)=\sigma_N\sqrt{\pi a}\cdot F_N(\xi) \tag{18}$$

where F_N is the edge-cracked strip's correction factor for finite width and the one we employed here is as follows (Tada et al., 1985).

$$F_N(\xi)=\sqrt{\frac{2}{\pi\xi}\tan\frac{\pi\xi}{2}}\cdot\frac{0.752+2.02\xi+0.37\left[1-\sin(\pi\xi/2)\right]^3}{\cos(\pi\xi/2)} \tag{19}$$

We conclude from Fig. 6 that the weight function given by Eq. (15) can evaluate the SIF of the circumferential crack in a cylinder under uniform stress distribution on the crack surface with sufficient practical accuracy, including the effect of cylinder length.

4. 3 Quadratic stress distribution

As the last problem we consider the SIF for the case in which the surface of the circumferential crack in a cylinder is subjected to a quadratic stress distribution of $\sigma(x) = 98\ (1-x/W)^2$ MPa. We compare the SIF K_{weight}, which we evaluated by Eq. (15), with the SIF K_{FEM}, which we evaluated from the FEM result. We normalized the calculated results by $K_{Labbens}$, which is the SIF evaluated by applying Labbens' weight function (Labbens, 1976), and showed it in Fig. 7. Note that $K_{Labbens}$ corresponds to the SIF of the crack in a long cylinder.

We conclude from Fig. 7 that the weight function given by Eq. (15) can evaluate the SIF of the circumferential crack in a cylinder under quadratic stress distribution on the crack surface with sufficient practical accuracy, including the effect of cylinder length.

5. CONCLUSIONS

In this paper, we derived the weight function of a circumferential crack in a cylinder subjected to an arbitrary stress distribution on the crack surface, which can easily include the effect of the structural parameters such as cylinder length on the SIF. In this process, we also derived the closed form equation to evaluate the crack opening displacement of the configuration subjected to a linear stress distribution on the crack surfaces.

Subsequently, we evaluated the SIFs of the crack subjected to linear, uniform and quadratic stress distributions on the crack surfaces, by applying the derived weight function, and confirmed the validity of the weight function by comparing the SIF with the corresponding solution obtained by other methods, such as FEM. The results show that the cylinder length affects the SIF for all three cases and should be properly treated.

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APPENDIX

We previously derived the closed form equation to evaluate the SIF for an arbitrarily located circumferential crack in a thin-walled cylinder under linear stress distribution on crack surface as follows (Meshii and Watanabe, 1998b).

$$K_{M0}=\left(\frac{M_C}{M}\right)\cdot\frac{1}{\phi_a}\cdot\left[\frac{M_0}{Z}\sqrt{\pi a}\cdot F_M(\xi)\right] \tag{A 1}$$

where (M_C/M) and ϕ_a are structurally determined parameters, given as follows.

$$\phi_a=\frac{\sinh\beta h_1\cos\beta h_2+\cosh\beta h_1\sin\beta h_2+\sinh\beta h_2\cos\beta h_1+\cosh\beta h_2\sin\beta h_1}{\sinh\beta H+\sin\beta H} \tag{A 2}$$

$$\begin{aligned}\frac{M_C}{M}=&\\ &\{2(\cos\beta h_1\cosh\beta h_1+\cos\beta h_2\cosh\beta h_2)\\ &+\sin\beta(H+h_1)\sinh\beta h_2-\sin\beta h_2\sinh\beta(H+h_1)+\sin\beta(H+h_2)\sinh\beta h_1-\sin\beta h_1\sinh\beta(H+h_2)\\ &-\cos\beta(H+h_1)\cosh\beta h_2-\cos\beta h_2\cosh\beta(H+h_1)-\cos\beta(H+h_2)\cosh\beta h_1-\cos\beta h_1\cosh\beta(H+h_2)\}\\ &/[2-\cos 2\beta H-\cosh 2\beta H+\\ &\beta D\cdot\Delta\lambda\{\sin 2\beta H-\sinh 2\beta H+2(\sinh 2\beta h_1-\sin 2\beta h_1)+2(\sinh 2\beta h_2-\sin 2\beta h_2)\\ &+\sin 2\beta h_1\cosh 2\beta h_2-\cos 2\beta h_1\sinh 2\beta h_2+\sin 2\beta h_2\cosh 2\beta h_1-\cos 2\beta h_2\sinh 2\beta h_1\}]\end{aligned} \tag{A 3}$$

Here, h_1 and h_2 are the distances to the crack position from the top and the bottom of the cylinder, respectively, and $h_1 + h_2 = H$. The constant β used in these equations is a parameter for replacing a cylindrical shell with a beam on an elastic foundation. The definition of β is as follows (Timoshenko, 1934).

$$\beta=\sqrt[4]{\frac{3(1-\nu^2)}{R_m^2W^2}} \tag{A 4}$$

Here, R_m : mean radius, W: thickness, ν: Poisson's Ratio.

We obtain $\psi_f(\xi, \beta H)$ (= $(M_C/M)/\phi_a$) by applying these equations for the case shown in Fig. 2, that is, by substituting $h_1 = h_2 = H/2$ as

$$\psi_f(\xi, \beta H) = \frac{\sinh \beta H + \sin \beta H}{\sinh \beta H + \sin \beta H + \beta D \cdot \Delta\lambda(\xi) \cdot (\cosh \beta H + \cos \beta H - 2)} \tag{A 5}$$

LIST OF FIGURES

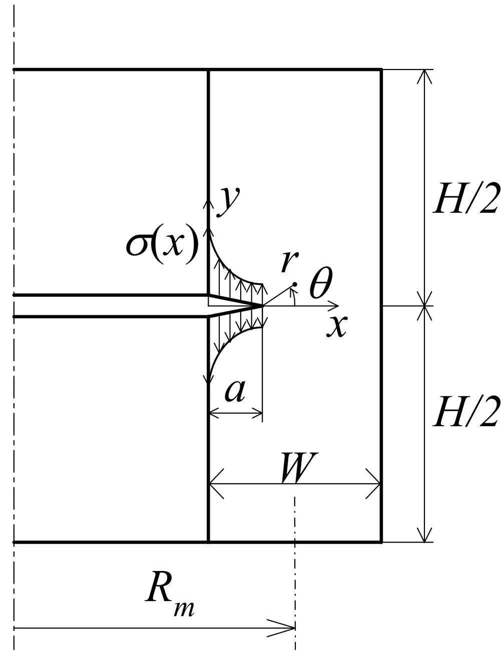

Fig. 1 A cylinder with a circumferential crack under arbitrary stress distribution

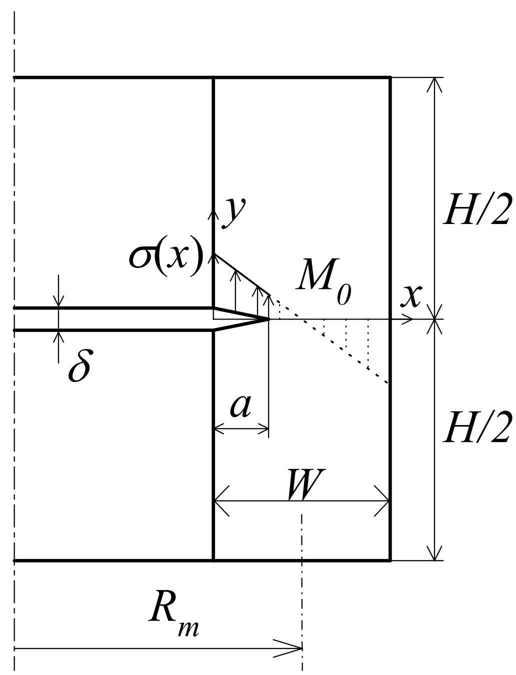

Fig. 2 A cylinder with a circumferential crack under linear stress distribution

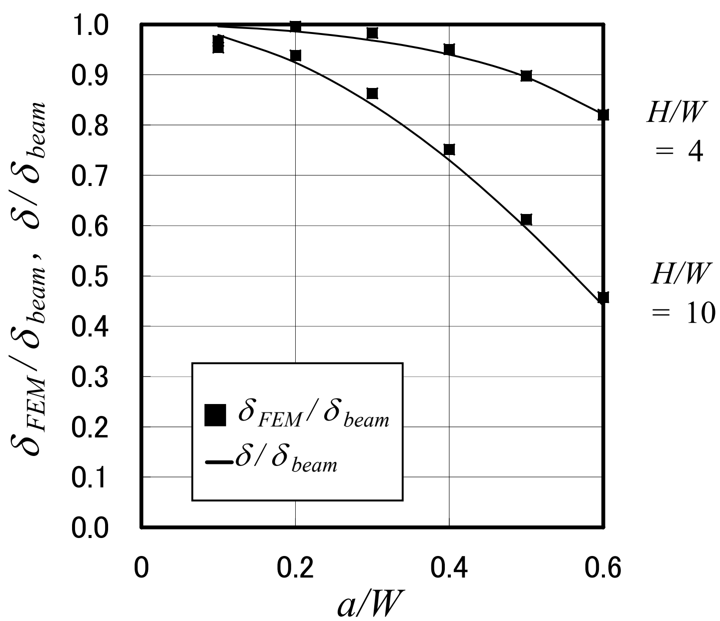

Fig. 3 Crack opening displacement at the crack end subjected to linear stress distribution

（R_m/W = 10.5、 W = 10 mm）

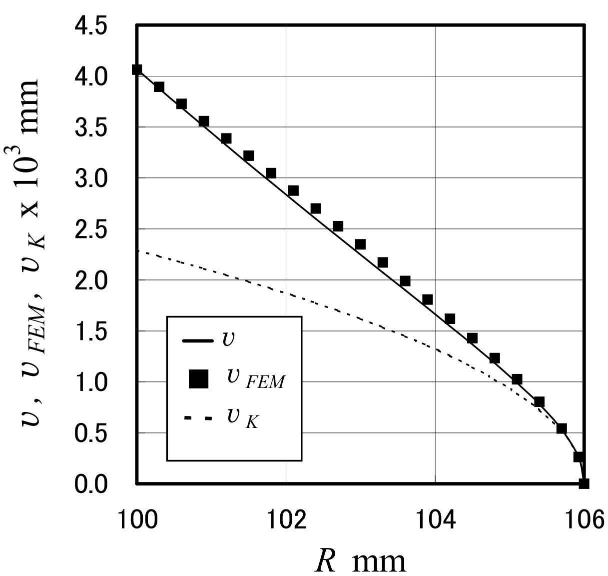

Fig. 4 Crack opening displacement along the crack surface subjected to linear stress distribution

（R_m/W = 10.5、H/W = 4、a/W =0.6、W = 10 mm ）

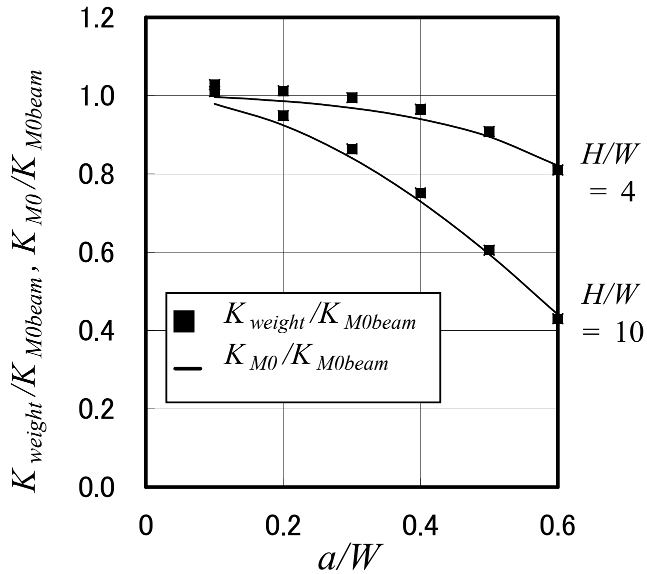

Fig. 5 Comparison of the SIF under linear stress distribution ($R_m/W = 10.5$)

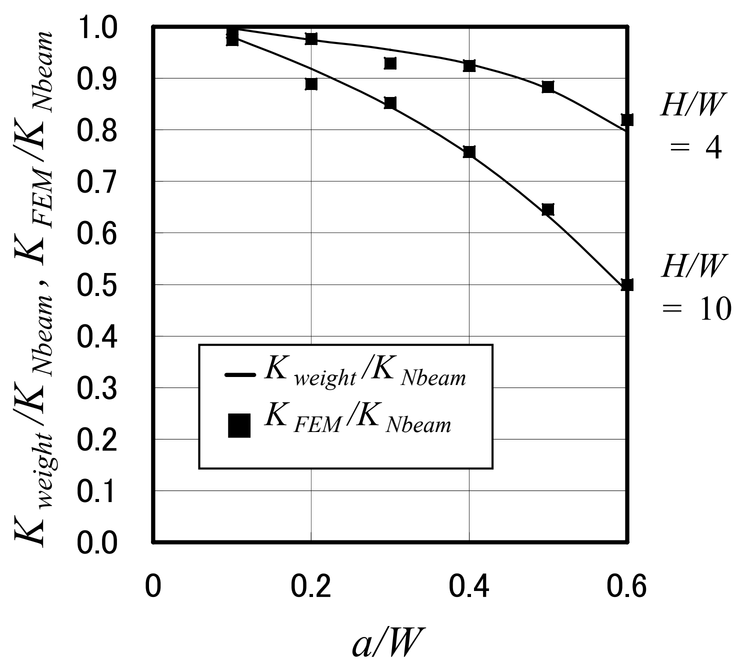

Fig. 6 Comparison of the SIF under uniform stress ($R_m/W = 10.5$)

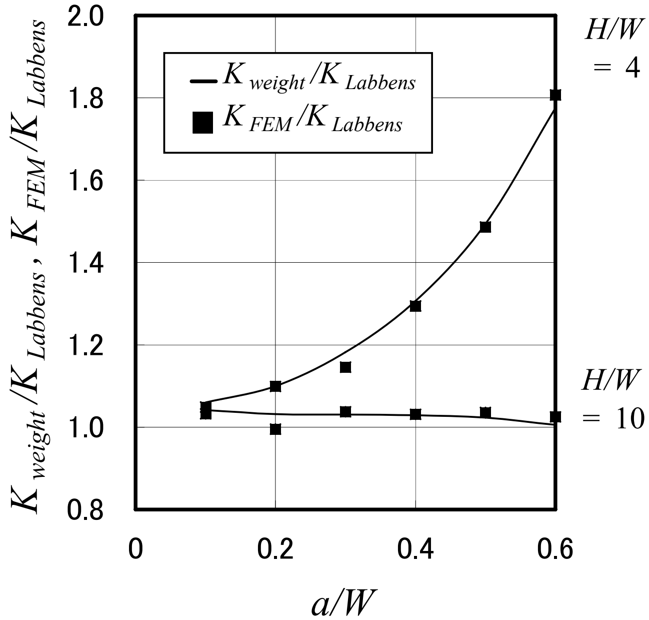

Fig. 7 Comparison of the SIF under quadratic stress distribution (R_m/W = 10.5)