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1 Linear perturbation

The Lagrangian description for the cosmological fluid can be usefully applied to the structure formation scenario. This description provides a relatively accurate model even in a quasi-linear regime. Zel'dovich [**?**] proposed a linear Lagrangian approximation for dust fluid. This approximation is called the Zel'dovich approximation (ZA) [**?**, **?**, **?**, **?**].

In ZA and its extended models, pressure was ignored. Recently, Lagrangian approximation in which the effect of pressure was taken into consideration have been analyzed. Buchert and Domínguez [**?**] discussed the effect of velocity dispersion using the collisionless Boltzmann equation They argued that models of a large-scale structure should be constructed for a flow describing the average motion of a multi-stream system. Then they showed that when the velocity dispersion is regarded as small and isotropic it produces effective "pressure" or viscosity terms. Furthermore, they derived the relation between mass density ρ and pressure P, i.e., an "equation of state." Hereafter, we call this model the Euler-Jeans-Newton (EJN) model. Actually, Adler and Buchert [**?**] have formulated the Lagrangian perturbation theory for a barotropic fluid. Morita and Tatekawa [**?**] derived the linear perturbative solutions for the polytropic fluid in Einstein-de Sitter Universe model. Then Tatekawa *et al.* [**?**] showed the solutions in generic Friedmann Universe models.

In the Lagrangian approximation, the displacement of the fluid element from homogeneous distribution is regarded as a perturbative quantity. Using this formalism, the matter density is described with exact form. Furthermore we can obtain relatively good description for the density field, because the relation between the density and the displacement is nonlinear.

2 Higher-order approximation

ZA solutions are known as perturbative solutions, which describe the structure well in the quasi-nonlinear regime. To improve approximation, higher-order perturbative solutions of Lagrangian displacement were derived. For the EJN model, the higher-order perturbative solutions are derived for only Einstein-de Sitter Universe model.

Table 1 shows the references which derive higher-order perturbative solutions. For other extension which improves Lagrangian description, Tatekawa [?] classified them.

When we continue applying the solutions of ZA or its higher-order approximation after the appearance of caustics, the nonlinear structure diffuses and breaks. For avoidance of the formation of caustics, several modified models have been proposed. For detail, Tatekawa [?] mentioned them.

Table 1. Higher-order Lagrangian perturbative solutions

order	dust model	the EJN model
2nd	Bouchet *et al.* [?]	Morita and Tatekawa [?]
	Buchert and Ehlers [?]	Tatekawa *et al.* [?]
3rd	Buchert [?]	Tatekawa [?]
	Bouchet *et al.* [?]	Tatekawa [?]
	Catelan [?]	
	Sasaki and Kasai [?]	

3 The validity of Lagrangian description

In Sec. 1, we mentioned that the Lagrangian description provides a relatively accurate model even in a quasi-linear regime. Here we check the validity of the Lagrangian description.

We consider the development of the spherical void with Lagrangian perturbation. Here we consider "top-hat" spherical void, i.e., a constant density spherical void in Einstein-de Sitter Universe.

Munshi, Sahni, and Starobinsky [?] derived up to the third-order perturbative solution. In addition to these, Sahni and Shandarin [?] obtained up to fifth-order, Tatekawa [?] obtained up to eleventh-order. They concluded that ZA remains the best approximation to apply to the late-time evolution of voids. From the viewpoint of the convergence of the series, the conclusion seems reasonable.

4 Future prospect

According to past study, it is well-known that the Lagrangian description for cosmic fluid realizes quasi-nonlinear structure well. Why is the Lagrangian description so accurate? For proper reason of this problem, several studies have been carried out. Recently, Yoshisato *et al.* [**?**] discussed the reason. They have argued that the Lagrangian description extracts the essence of the gravitational collapse.

Recently, Buchert and Domínguez [**?**] proposed systematic derivation of the equation of motion for cosmic fluid. They propose systematic generalizations for Newtonian evolution equations. Then they discuss some non-perturbative results for structure formation and try to clarify the phenomenon of stabilization of large-scale structure emerging from gravitational instability. For reasonable description for high dense region or multi-stream region, we should study their generalized approaches.

The Lagrangian perturbation theory seems rather useful until quasi-nonlinear regime develops. However, the density fluctuation becomes strongly nonlinear in the present Universe. Can we apply the theory to the problem of structure formation? Because a huge simulation has executed [**?**], someone may thinks that the perturbation theory is no longer useful.

One possibility is analysis of the past structure. Because the density contrast will still be small in the high-z region, we expect that we will be able to discuss the characters of the density fluctuation using the Lagrangian description well.

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