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Abstract

The fast cyclotron wave excitation resulting from the coupling with the electron Bernstein wave is observed in a magnetized plasma penetrated by a spiral electron beam. For the excitation to occur, the beam energy component perpendicular to the magnetic field is larger than a critical value. From measurements of the wave number and the growth rate of the excited cyclotron wave, its dispersion relation is determined for various values of beam and plasma parameters. The experimental results are compared with the theoretical considerations.

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1 Introduction

The instability of the wave in a beam-plasma system and its nonlinear development have been investigated with a great interest by many authors.[1),2)] For the weak beam interaction, the wave becomes unstable near the frequencies of beam waves, that is, $\omega = k_{\parallel} v_{0\parallel} + n\omega_c$, where $k_{\parallel}$ and $v_{0\parallel}$ are the parallel components of wave vector and beam velocity and $\omega_c/2\pi$ is the cyclotron frequency. In the case where an electron beam is injected parallel to the external magnetic field into a plasma, the space charge wave (n=0) and the slow cyclotron wave (n=-1) of beam have the negative energy and become unstable, as the result of the coupling with the plasma wave.[3)-5)] These are called the Cherenkov excitation and the slow cyclotron wave excitation which are the most dominant process in the system.

On the other hand, in the case where a beam is injected obliquely to the external field and has the energy component $E_{\perp}$ perpendicular to the field, the fast cyclotron wave (n=1) of beam may also have the negative energy and be excited, which is called the fast cyclotron wave excitation.

In this paper, it is reported that the latter excitation is observed when $E_{\perp}$ is larger than a critical value which is about 20 percent of the total beam energy E_b $(= E_{\perp} + E_{\parallel})$. Then, Cherenkov excitation can always occur together with fast or slow cyclotron wave excitation, but both fast and slow cyclotron wave excitations do not occur at the same time. For the larger value of $E_{\perp}$, higher modes of beam waves ($|n| \geqq 2$) are excited. These results can be explained consistently by the theoretical considerations following Seidl's paper,[6] which is given in § 2. In § 3, the experimental apparatus and procedures are explained. In § 4, the experimental results and discussions are given, and in § 5, the concluded remarks are descibed.

2 Theoretical consideration

Following Seidl's paper,[6] we describe in brief the theoretical consideration for the instability of wave in the system where the magnetized Maxwellian plasma is penetrated by the monoenergetic spiral electron beam.

2.1 Dispersion relation

The dispersion relation of the wave in the system is given by using the susceptibilities (ε_p and ε_b) of plasma and beam, as follows,

$$1 + \varepsilon_p + \varepsilon_b = 0. \tag{1}$$

As well-known, ε_p is expressed for the Maxwellian plasma, as follows,

$$\varepsilon_p = \frac{\omega_p^2}{k^2 v_t^2}\left[1 + \sum_{n=-\infty}^{\infty} \exp(-\lambda)\, I_n(\lambda) \frac{\omega}{\sqrt{2}\, k_{\parallel} v_t} z\left(\frac{\omega - n\,\omega_c}{\sqrt{2}\, k_{\parallel} v_t}\right)\right], \tag{2}$$

where ω_p is the plasma frequency, v_t is the thermal speed of plasma electrons, $\lambda = \frac{1}{2}(k_{\perp} v_t/\omega_c)^2$, $k_{\perp}$ is the perpendicular component of wave vector, $I_n(\lambda)$ are modified Bessel functions of the first kind and Z(x) is the plasma dispersion function.[7)]

If the velocity distribution of beam is assumed as follows,

$$f_b = \frac{1}{2\pi v_\perp} \delta(v_\perp - v_{0\perp}) \, \delta(v_\parallel - v_{0\parallel}), \tag{3}$$

then,

$$\varepsilon_b = -\frac{\omega_b^2}{\omega_c^2} \sum_{n=-\infty}^{\infty} \left[\frac{S_n \omega_c^2}{(\omega - n\omega_c - k_\parallel v_{0\parallel})^2} \frac{k_\parallel^2}{k^2} + \frac{n\, T_n \omega_c}{\omega - n\omega_c - k_\parallel v_{0\parallel}} \frac{k_\perp^2}{k^2} \right], \tag{4}$$

where $v_\perp$ is the perpendicular component of velocity, ω_b is the plasma frequency of electron beam, $S_n = J_n^2(\mu)$, $T_n = \frac{2}{\mu} J_n(\mu)\, J_n'(\mu)$, $\mu = k_\perp v_{0\perp}/\omega_c$ and $J_n(\mu)$ are Bessel functions.

2.2 Criterion for the wave excitation

Here, we consider only the weak beam interaction with plasma ($\omega_b \ll \omega_p$). Then, the dispersion equation (eq. (1)) can be expanded around the intersection point $\omega = \omega_0$ of both dispersion relations of the nth mode of beam wave ($\omega - n\omega_c - k_\parallel v_{0\parallel} = 0$) and the plasma wave ($1 + \varepsilon_p = 0$), and approximated by using the dimensionless small parameter $\delta = (\omega - \omega_o)/\omega_c$, as follows,

$$\delta^3 - \left(\frac{\omega_b^2}{\omega_c^2} g^2 \, G \, n \, T_n\right) \delta - \left(\frac{\omega_b^2}{\omega_c^2} (x - n)^2 \, G \, S_n\right) = 0, \qquad (5)$$

where $g = k_\perp v_{0\parallel} / \omega_c$, $x = \omega_0 / \omega_c$ and $G = \frac{k^2 v_{0\parallel}^2}{\omega_c} \left.\frac{\partial \varepsilon_p}{\partial \omega}\right|_{\omega=\omega_0}$.

This equation will yield two complex roots, one corresponding to instability ($\mathrm{Im}\,\delta > 0$), if

$$\left[(x - n)^2 \, S_n\right]^2 > \frac{4}{27}\left(\frac{\omega_b}{\omega_c}\right)^2 G \, (g^2 \, m \, T_m)^3. \qquad (6)$$

Under the cold plasma assumption, i.e.,

$$\varepsilon_p = - \frac{\omega_p^2}{\omega^2} \frac{k_\parallel^2}{k^2} - \frac{\omega_p^2}{\omega^2 - \omega_c^2} \frac{k_\perp^2}{k^2},$$

G is calculated as follows,

$$G = \frac{x}{2} \cdot \frac{x^2 - 1}{(x^2 - 1)\,(x - n)^2 + g^2 x^4} \cdot \frac{(x - n)^2 (x^2 - 1) + g^2 x^2}{(x - n)^2 + g^2} \qquad (7)$$

Since $G > 0$ for all values of x (> 1), the sufficient condition for eq. (6) is

$$n \, T_n \leqq 0. \qquad (8)$$

In Fig.1, $-T_n$ are shown as functions of μ. From the wave excitation condition (eq. (8)) and this figure, we can remark as follows, 1. Cherenkov excitation (n=0)

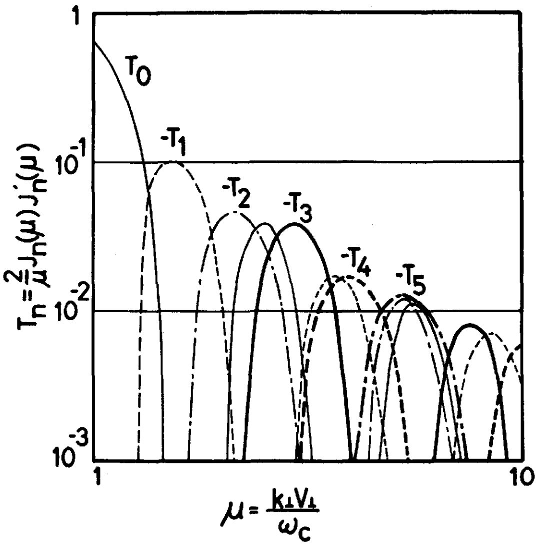

Fig. 1 $-T_n$ are plotted as functions of $\mu = k_\perp v_{0\perp}/\omega_c$. The fast cyclotron modes are unstable in the regions of μ where $-T_n$ has positive value, while the slow modes are unstable if $-T_n < 0$.

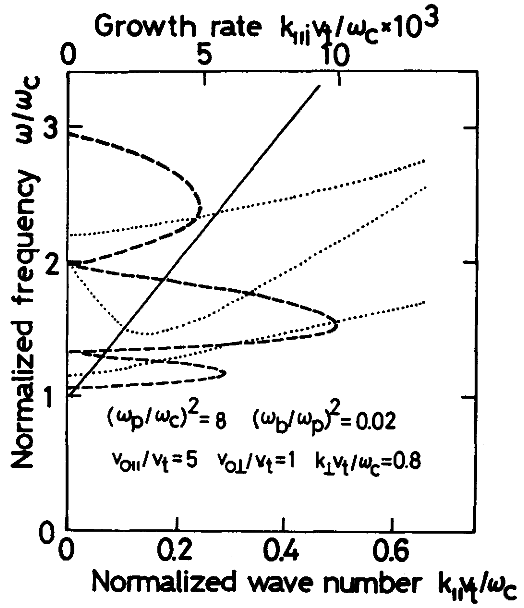

Fig. 2 The dispersion relation of the unstable fast cyclotron wave in a spiral electron beam-plasma system (solid and broken lines) and the stable electron Bernstein wave in a plasma. The maximum growth rates ($k_{\parallel i}$) occur at the intersection points of both dispersion curves.

occurs for all values of $E_\perp$. 2. Fast mode excitations ($n \geqq 1$) occur when T_n becomes negative for sufficiently large value of $E_\perp$. 3. Since $T_{-n}=T_n$, slow mode excitations ($n \leqq -1$) can occur even for $E_\perp=0$, that is, for the case of parallel beam injection. For the spiral beam, both fast and slow mode excitations of same mode number $|n|$ do not occur simultaneously.

2.3 Numerical solution for the dispersion relation

Under the condition where fast cyclotron wave excitation does occur, that is, T_1 has negative value, the dispersion relation (eq. (1)) is analyzed numerically, assuming that $k_{\parallel}$ is complex. The result is shown in Fig. 2. The solid and broken lines show the real and imaginary parts of $k_{\parallel}$ for unstable fast cyclotron wave, while dotted lines do the dispersion relation of the plasma wave (the electron Bernstein wave) in the absence of the beam ($1 + \varepsilon_p = 0$). It is seen that the imaginary part ($k_{\parallel i}$) has the maximum values, near the intersection points ($\omega = \omega_0$) of both dispersion relation curves $\omega - k_{\parallel} v_{0\parallel} - \omega_c = 0$ and $1 + \varepsilon_p = 0$. In conclusion, the instability results from the coupling of the fast cyclotron wave of beam with the electron Bernstein wave of plasma.

3 Experimental apparatus and procedures

In order to investigate the propagation of waves and their instability due to the interaction of an electron beam with a plasma, it is desired that a Maxwellian plasma is produced and an electron beam is injected into this plasma, parameters of beam being varied independently on those of plasma. Considering such a requirement, we have set up the apparatus (Fig. 3), which is consisted of three regions, that is, the dc discharge region, the plasma diffused region (or the region of the beam-plasma system) and the beam-generated region. This apparatus has be shown in a previous paper.[3)] The plasma produced by dc discharge in Ar gas (pressure $p_1 = 1 - 2 \times 10^{-2}$ Torr) is diffused into the second region ($p_2 = 1 - 2 \times 10^{-3}$ Torr) along the line of magnetic force. An electron beam is produced by the Pierce gun in the third region ($p_3 = 0.8 - 1.0 \times 10^{-4}$ Torr), and injected into the second region. The ratio of the beam energy component to the total energy $E_\perp/E_b$ can be varied continuously by varying the beam injection angle θ with repect to the line of force. The parameters of the beam-plasma system are as follows, the plasma density $n_p = 5 \times 10^8 - 8 \times 10^{10}$ cm^{-3}, the electron temperature $kT_e = 5 - 10$ eV, the beam density $n_b = 1.5 - 4.5 \times 10^8$ cm^{-3}, the beam temperature $kT_b = 0.3$ eV, the total beam energy $E_b = 100 - 350$ eV, the beam energy component $E_\perp = 0 - 80$ eV, and the electron cyclotron freauency $\omega_c/2\pi = 308$ MHz.

The test wave is excited by the coaxial probe situated in the center of the beam-plasma system (z=0) and detected by the other probe movable axially. By using the interferometer system, the propagating wave patterns along the field (along the axial direction z) are observed.

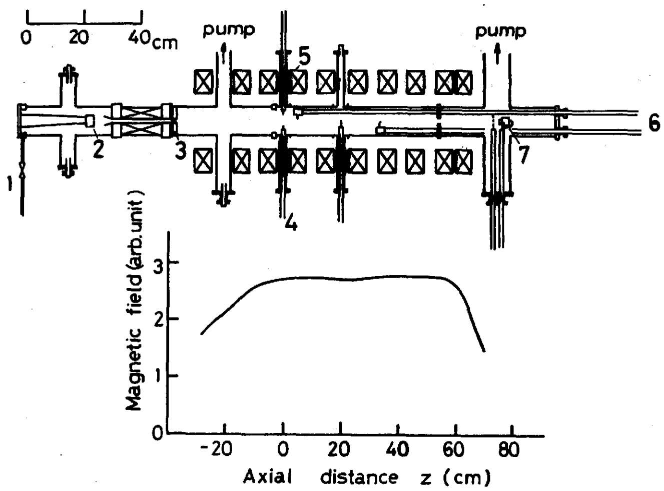

Fig. 3 The experimental apparatus and the distribution of external magnetic field. 1. gas inlet, 2. cathode, 3, anode, 4. r-probe, 5. coils, 6. z-probe and 7. electron gun.

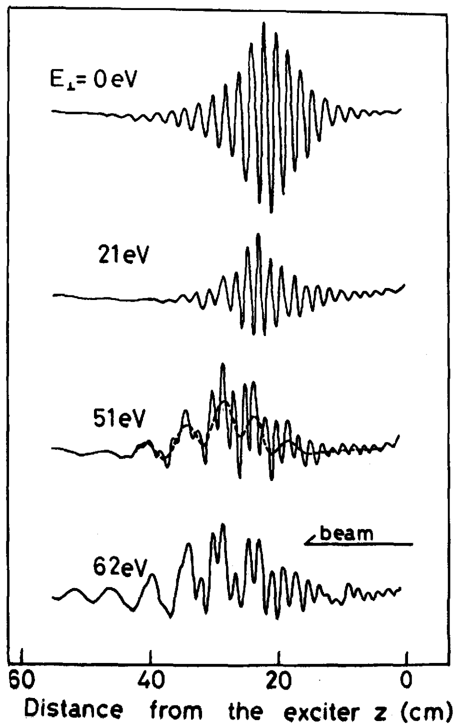

Fig. 4 The propagating wave patterns. For $E_{\perp}$= 51 eV and 62 eV, the excitation of the fast cyclotron wave is seen.

The parallel component of beam velocity $v_{0\parallel}$ is determined from the pitch p of spiral motion, which is measured by the spatial variation of probe current.

$$v_{0\parallel} = p\ \omega_c/2\pi . \qquad (9)$$

4 Experimental results and discussions

4.1 Excitation of fast cyclotron wave (n=1)

The propagating wave patterns measured by the interferometer system are shown in Fig. 4, with the beam energy component $E_\perp$ as a parameter. In the case of $E_\perp = 0$ eV which is shown in the upper trace, only the Cherenkov excitation (n=0) can occur and the space charge wave of beam grows along the direction of streaming of electron beam. The wave number $k_{\parallel 0}$ of growing wave satisfies the relation $\omega = k_{\parallel 0} v_{0\parallel}$, This result is the same as that of the previous paper.[3] On the other hand, for the sufficiently large value of $E_\perp$, the other wave of smaller wave number $k_{\parallel 1}$ (as shown by the dotted line) is seen, overlaping on the space charge wave of wave number $k_{\parallel 0}$. The former wave may be considered to be the fast cyclotron wave of beam.

4.2 Dispersion relation of the wave

The similar wave patterns are observed for the fixed value of $E_\perp$ (= 52 eV), with the exciting frequency $\omega/2\pi$ as a parameter. The wave numbers ($k_{\parallel 0}$ and $k_{\parallel 1}$) of both waves and the amplification factor $k_{\parallel i}$ for the wave of smaller wave number $k_{\parallel 1}$ are estimated from the patterns, They are plotted as functions of $\omega/2\pi$ in Fig. 5, which

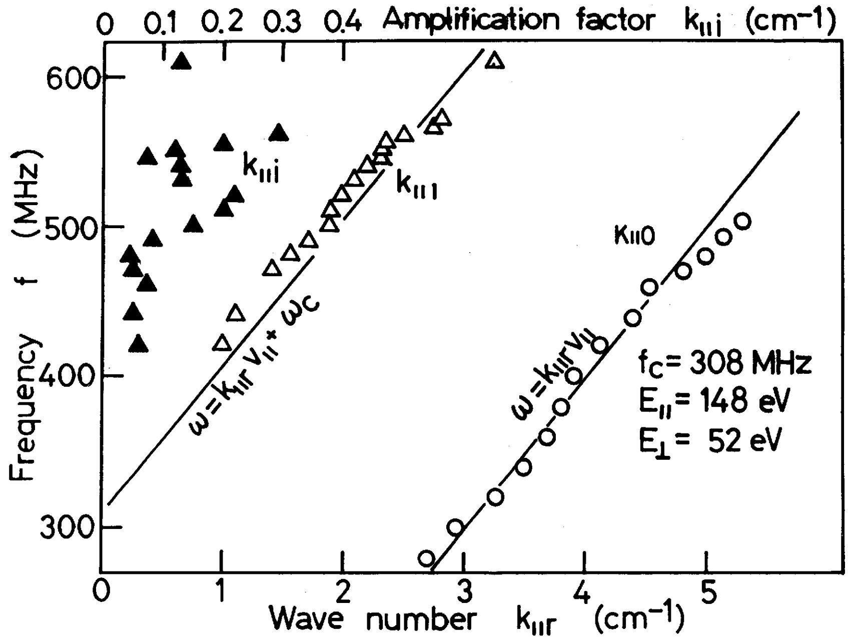

Fig. 5 Observed wave numbers $k_{\|0}$ and $k_{\|1}$ and growth rate $k_{\|i}$ corresponding to $k_{\|1}$ are plotted as functions of $\omega/2\pi$.

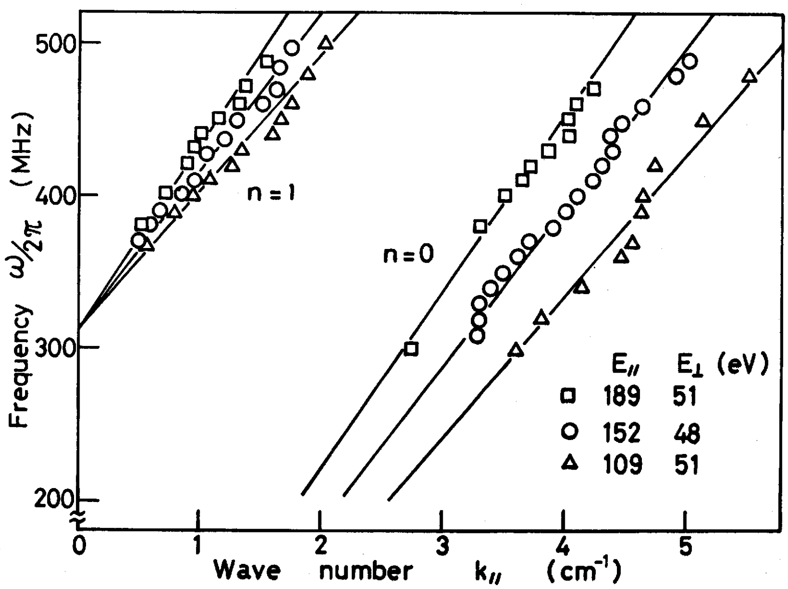

Fig. 6 Observed wave numbers $k_{\|0}$ and $k_{\|1}$ are plotted as functions of $\omega/2\pi$ with $v_{0\|}$ as a parameter.

shows the dispersion relation of both waves. It is seen that $k_{\parallel 1}$ satisfies the excitation condition of the fast cyclotron wave $\omega = k_{\parallel} v_{0\parallel} + \omega_c$, though $k_{\parallel 0}$ does the Cherenkov excitation condition $\omega = k_{\parallel} v_{0\parallel}$.
Comparison of the results with the theoretical consideration shown in Fig. 2, suggests that the observed excitations result from the coupling of the space charge or the fast cyclotron waves of beam with the electron Bernstein wave.

The similar experiments are done for various values of the parallel beam velocity $v_{0\parallel}$ and the electron cyclotron frequency $\omega_c/2\pi$. Observed wave number $k_{\parallel 0}$ and $k_{\parallel 1}$ of both waves are plotted as functions of $\omega/2\pi$, in Figs. 6 and 7. $k_{\parallel 0}$ and $k_{\parallel 1}$ always satisfy the relations mentioned above, when $v_{0\parallel}$ and ω_c are varied. These facts support the explanation that the observed wave of the wave number $k_{\parallel 1}$ is the unstable fast cyclotron wave.

4.3 Criterion for the fast cyclotron wave excitation

For the various values of beam parameters $E_{\perp}$ and E_b, the excitation of the wave is tried, the results of which are shown in Fig. 8. Solid circles show the occurence of the excitation of fast cyclotron wave. When the ratio $E_{\perp}/E_b$ is larger than about 20 percent, the excitation does occur. This fact is consistent with the theoretical consideration given in § 2, where an electron beam is assumed to be the monoenergetic spiral one. It is shown that the wave (n=1 mode) becomes unstable, if $T_1 \leqq 0$. The bands of the wave excitation, that is, the regions

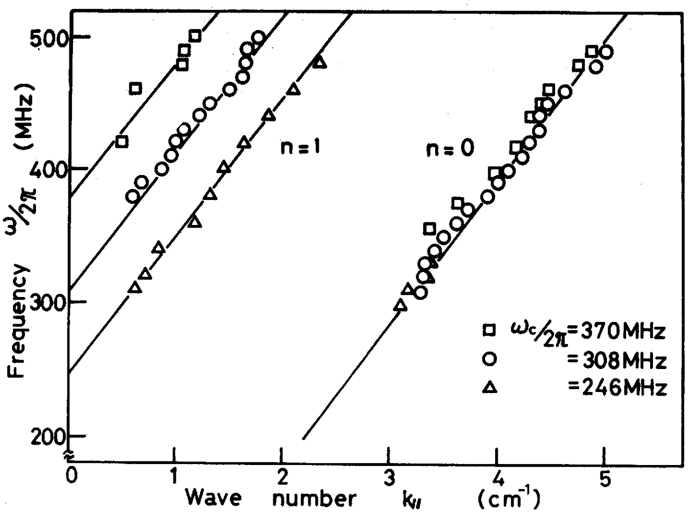

Fig. 7 Observed wave numbers $k_{\parallel 0}$ and $k_{\parallel 1}$ are plotted as functions of $\omega/2\pi$ with ω_c as a parameter.

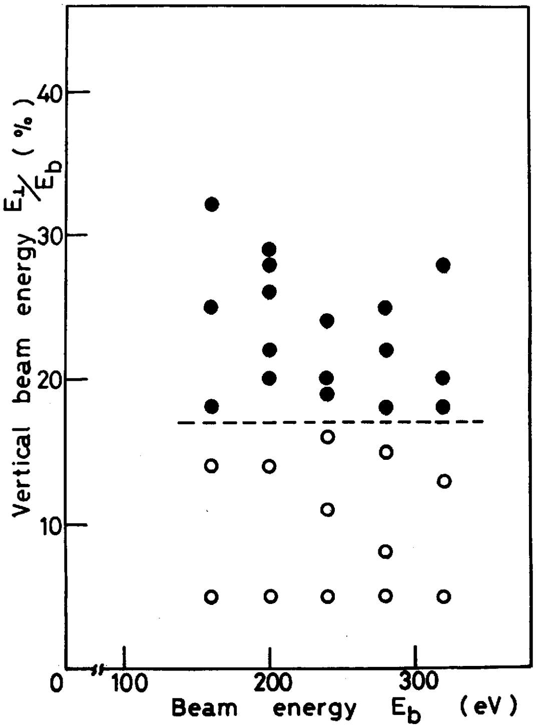

Fig. 8 The region in a beam parameter space ($E_\perp$-E_b space) where the fast cyclotron wave excitation occurs. Solid circles show the occurence of the excitation.

where T_l has negative value, are expressed by the following equation.

$$\nu_{lm} < k_\perp v_{0\perp} / \omega_c < \nu'_{lm} , \qquad (10)$$

where ν_{lm} and ν'_{lm} are the mth zeros of $J_l(\mu)$ and $J'_l(\mu)$.

On the other hand, under the experimental conditions, i.e., E_b= 240 eV, $E_\perp$ = 48 eV and $\omega_c/2\pi$ = 308 MHz, the wave patterns propagating radially are observed, with the wave frequency $\omega/2\pi$ as a parameter and the perpendicular wave number $k_\perp$ of the unstable fast cyclotron wave is determined. The value lies in the region of 10 $cm^{-1} < k_\perp <$ 20 cm^{-1}, which corresponds to the region of $1.72 < k_\perp v_{0\perp} / \omega_c < 3.43$. This region consists approximately with the first band (m=1) for wave excitation condition, which is the reasonable result.

4.4 Observation of the slow cyclotron wave (n= -1) and the second mode (n= -2)

The slow cyclotron modes ($n \leq -1$) are unstable and excited for the inverse regions of those denoted by eq. (10). Therefore, they should be observed even for the parallel injection of beam ($E_\perp$ = o eV). However, the Cherenkov excitation is so intense that they cannot be observed for the first band ($k_\perp v_{0\perp} / \omega_c < \nu_{n1}$).

When $E_\perp$ is increased and the second band of excitation condition for the slow cyclotron mode ($\nu'_{n1} < k_\perp v_{0\perp} / \omega_c < \nu_{n2}$) is attained, the slow cyclotron wave (n= -1) and the second mode (n= -2) are observed to be excited, overlaping on the rather weak excitation of the space charge wave (n=0).

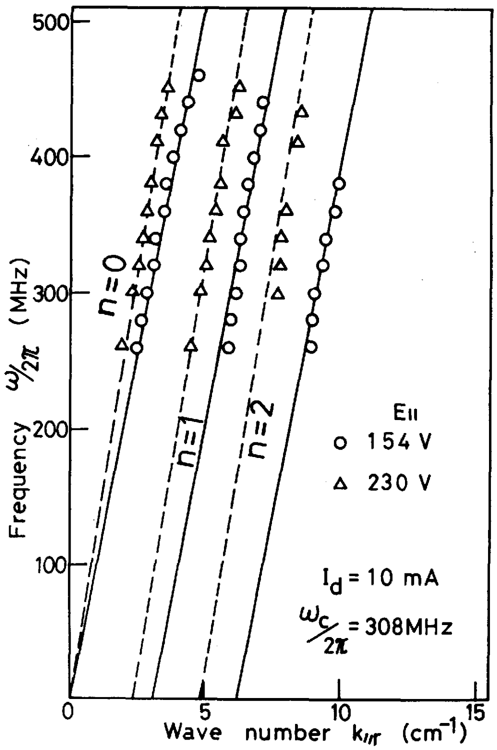

Fig. 9 Observed dispersion relation of the slow cyclotron modes and the space charge wave, with $v_{0\parallel}$ as a parameter.

The measured wave numbers $k_{\parallel}$ are plotted as functions of $\omega/2\pi$ with the parallel velocity component of beam in Fig. 9. The solid and broken lines show the dispersion relations of beam waves calculated by using the experimental conditions. The experimental results are in fairly good agreement with the calculated curves.

5 Conclusion

It is concluded that the fast cyclotron wave (n=1) of beam has negative energy and becomes unstable as the result of coupling with the electron Bernstein wave, when the spiral electron beam is injected into the plasma. The experimentally obtained dispersion relation of the unstable wave is in agreement with the theoretical one. The threshold value of beam energy component $E_{\perp}$ perpendicular to the external field for the instability, is studied experimentally and compared with the theoretical result. Both coinside with each other. For larger value of $E_{\perp}$, the excitation of higher modes of beam waves are excited.

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