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journal or publication title	福井大学工学部研究報告
volume	25
number	1
page range	61-76
year	1977-03
URL	http://hdl.handle.net/10098/4478

Ion Bernstein Wave in an Ion Beam-Plasma System

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(Received Nov. 29, 1976)

The dispersion relation of ion Bernstein wave is analyzed numerically, for a Maxwellian plasma and an ion beam-plasma system. In an ion beam-plasma system, the coupling of the wave with the slow space charge wave and the slow cyclotron wave results in the electrostatic instability, while that with the fast space charge wave and the fast cyclotron wave does in a strong damping of the wave. The behaviors of the wave are studied in detail for both cases of a Maxwellian plasma and an ion beam-plasma system, the results of which are compared with the electron Bernstein wave.

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1 Introduction

The Bernstein wave is the most general, electrostatic wave in a magnetized plasma, which appears as the so-called cyclotron harmonics in the experimental study.¹⁻⁵ The investigation of the electron Bernstein wave (electron cyclotron harmonics) has been done with great interest by many authors,⁶⁻¹⁶ with respect to the propagation characteristics in a Maxwellian plasma, an electrostatic instability in an electron beam-plasma system and its nonlinear development. They show that the wave does propagate as a forward wave and/or a backward wave, the former becoming convectively unstable but the latter absolutely unstable in a beam-plasma system, and the growth of resulting unstable wave saturates as the result of relaxation of beam energy.¹⁷⁻²⁰

On the other hand, the ion Bernstein wave has not been studied without a few works,²¹⁻²⁵ because of the difficulty of excitation of the wave. Schmitt has observed the propagation characteristics of the wave and shown that the wave propagates stably perpendicular to the magnetic field (which is named 'pure ion Bernstein wave') and its feature is very similar to the perpendicularly propagating electron Bernstein wave, while for a slightly oblique propagation, thermal electron whose velocities are comparable with the parallel velocity component of the wave, affect its propagation characteristics and the feature becomes appreciably different from that of the electron Bernstein wave. (which is named 'neutralized ion Bernstein wave')

The excitation of the wave due to the injection of ion beam has been examined by a few authors.^{26,27} One of them suggests that the wave excited in the frequency region between successive ion cyclotron harmonics are referred to the electrostatic instability in a beam-plasma system. However, the systematic analysis of the instability has not done as well as that of the propagation characteristics of the wave.

In this paper, the detailed informations for the wave, especially the propagation characteristics and the behavior of the instability are given, by solving numerically the dispersion relation in a Maxwellian plasma and in an ion beam-plasma system. In section 2, the dispersion relation is described. The results of calculation and the discussions for them are given in section 3, and they are summarized in brief in section 4.

2 Dispersion relation of ion Bernstein wave

The behavior of ion Bernstein wave in a Maxwellian plasma is deduced from the dispersion relation of an electrostatic wave propagating as an oblique angle, expressed as $\exp i(k_{\parallel}z + k_{\perp}x - \omega t)$,²⁸

$$\epsilon(\omega, k_{\parallel}, k_{\perp}) = 1 + \sum_{s=e,i} k_{Ds}^2/k^2 \exp(-\Lambda_s) \sum_{n=-\infty}^{\infty} I_n(\Lambda_s) \{1 + \xi_{0s} Z(\xi_{ns})\} = 0, \quad (1)$$

where $k^2 = k_{\parallel}^2 + k_{\perp}^2$, $k_{Ds}^2 = n_{0s} q_s^2 / \epsilon_0 k T_s = 2\omega_{ps}^2 / v_s^2$, $\omega_{ps}^2 = n_{0s} q_s^2 / \epsilon_0 m_s$, $\Lambda_s = k_{\perp}^2 \rho_s^2 / 2$, $\rho_s = v_s / \omega_{cs}$, $\omega_{cs} = q_s B_0 / m_s$, $v_s^2 = 2kT_s / m_s$, $\xi_{ns} = (\omega - n\omega_{cs}) / k_{\parallel} v_s$, Z is the plasma dispersion function,²⁹ v_s the thermal velocity and ρ_s the corresponding mean Larmor radius for species s . $\epsilon(\omega, k_{\parallel}, k_{\perp})$ is the dielectric constant for the wave.

For an ion beam-plasma system, a contribution of ion beam to the dielectric constant must be taken into account. If we assume that an ion beam is injected parallel to the magnetic field, the dispersion relation becomes as follows,

$$\epsilon'(\omega, k_{\parallel}, k_{\perp}) = \epsilon(\omega, k_{\parallel}, k_{\perp}) - \frac{\omega_b^2}{\omega_{ci}^2} \left(\frac{k_{\perp}^2}{k^2} \frac{\omega_{ci}^2}{(\omega - k_{\parallel} v_b)^2 - \omega_{ci}^2} + \frac{k_{\parallel}^2}{k^2} \frac{\omega_{ci}^2}{(\omega - k_{\parallel} v_b)^2} \right) = 0, \quad (2)$$

where ω_b is the plasma frequency of ion beam and v_b is the beam velocity.

Let's define the dimensionless quantities as follows,

$$T = T_i / T_e, \quad M = m_e / m_i, \quad V = v_b / v_i, \quad K_{\parallel} = k_{\parallel} v_i / \omega_{ci}, \quad K_{\perp} = k_{\perp} v_i / \omega_{ci}, \quad K^2 = K_{\parallel}^2 + K_{\perp}^2, \quad E = \omega_{ci}^2 / \omega_b^2,$$

$$P = \omega_{pi}^2 / \omega_{ci}^2 \text{ and } \Omega = \omega / \omega_{ci}.$$

Then the dispersion relations shown by eq.(1) and eq.(2) are rewritten by using these quantities, as follows,

$$\begin{aligned} \epsilon(\Omega, K_{\parallel}, K_{\perp}) = & 1 + 2 \frac{P}{K^2} \exp(-\frac{1}{2} K_{\perp}^2) \sum_{n=-\infty}^{\infty} I_n(\frac{1}{2} K_{\perp}^2) \left(1 + \frac{\Omega}{K_{\parallel}} Z\left(\frac{\Omega-n}{K_{\parallel}}\right)\right) \\ & + 2T \frac{P}{K^2} \exp(-\frac{1}{2} \frac{T}{M} K_{\perp}^2) \sum_{n=-\infty}^{\infty} I_n(\frac{1}{2} \frac{T}{M} K_{\perp}^2) \left(1 + \frac{\Omega}{(TM)^{1/2} K_{\parallel}} Z\left(\frac{\Omega-nM}{(TM)^{1/2} K_{\parallel}}\right)\right) = 0 \end{aligned} \quad (3)$$

and

$$\epsilon'(\Omega, K_{\parallel}, K_{\perp}) = \epsilon(\Omega, K_{\parallel}, K_{\perp}) - \frac{1}{B} \left(\frac{K_{\parallel}^2}{K^2} \frac{1}{(\Omega - K_{\parallel} V)^2} + \frac{K_{\perp}^2}{K^2} \frac{1}{(\Omega - K_{\parallel} V)^2 - 1} \right) = 0. \quad (4)$$

The numerical analyses for a Maxwellian plasma and an ion beam-plasma system are carried by using eqs. (3) and (4), respectively.

3 The results of calculations and discussions

3.1 Ion Bernstein wave in a Maxwellian plasma

The behavior of ion Bernstein wave propagating obliquely in a Maxwellian plasma is investigated by using eq.(3). Assuming that the normalized wave number components $K_{\parallel}$ and $K_{\perp}$ are real and the dimensionless quantities P , T and M are fixed at constant values, eq.(3) is solved numerically for complex frequency Ω . Here, the value of M is taken to be 1.4×10^{-4} , which shows that we treat He plasma.

The results of calculations are shown in Figs. 1 and 2. The dispersion relation surface in the frequency region $1 < \Omega < 2$ is shown in Fig. 1(a) and the corresponding damping rate Ω_i is shown in Fig. 1(b). For the Propagation

perpendicular to the magnetic field, i.e., $K_{\parallel}=0$, it is seen that the damping rate is vanished because resonant particles do not exist and the Landau damping does not occur. This is the case of the pure ion Bernstein wave, the feature of which is similar to that of the electron Bernstein wave propagating perpendicular to the magnetic field, as shown in the previous paper.³⁰ This result is also deduced immediately from eq. (3). For the case of $K_{\parallel}=0$, or more exactly, for the case of $\omega/k_{\parallel} \gg v_e$ (or $K_{\parallel} \ll \Omega(MT)^{1/2}$), the electron term vanishes in eq. (3), so that it is essentially equivalent to the dispersion relation of electron Bernstein wave, where the quantities for ion are replaced by those for electron.

For the slightly oblique propagation where the electron thermal velocity is comparable with the parallel phase velocity of wave, i.e., $\omega/k_{\parallel} \sim v_e$ (or $K_{\parallel} \sim \Omega(MT)^{1/2}$), resonant electrons exist and the wave is heavily Landau-damped, shown in Fig. 1(b). As seen in Fig. 1(a), the wave is backward for the wave number component $K_{\parallel}$, i.e., $\partial\Omega/\partial K_{\parallel} < 0$. These features are not found for the electron Bernstein wave, which has no particles corresponding to electrons of this case.

For more oblique propagation where the electron thermal velocity is much larger than the parallel phase velocity but the ion thermal velocity is much smaller than it, i.e., $v_i \ll \omega/k_{\parallel} \ll v_e$ (or $\Omega(MT)^{1/2} \ll K_{\parallel} \ll \Omega$), the damping rate is comparably small and the wave propagates stably. This is the case of the neutralized ion Bernstein wave.

In the case where the ion thermal velocity is comparable with or larger than the parallel phase velocity, i.e., $v_i > \omega/k_{\parallel}$ ($K_{\parallel} > \Omega$), it is seen in the figure that the damping rate becomes large because of the Landau damping due to thermal ions.

The dispersion relation in the frequency region $2 < \Omega < 3$ is calculated from

eq. (3) and shown in Fig.2 with $K_{\perp}$ as a parameter and for the constant values of P , T and M . It is known that the backward wave does not exist only in the region of small $K_{\parallel}$ but in the particular region of rather large $K_{\parallel}$. In the latter region where $\Omega/K_{\parallel}$ is about 3, the wave is the neutralized ion Bernstein wave, so that the damping rate is fairly small. Therefore, existence of the wave in the region suggests the occurrence of absolute instability due to the injection of ion beam.

3.2 The wave in an ion beam-plasma system

The behavior of the wave in an ion beam-plasma system is investigated by solving eq. (4). As well-known, the fast and the slow space charge waves and the fast and the slow cyclotron waves are present in a beam, whose dispersion relation are presented by the following equation by setting the plasma density P equal to zero in eq. (4)

$$\left(\frac{K_{\parallel}}{K_{\perp}}\right)^2 = - \frac{\Omega'^2 (\Omega'^2 - 1 - 1/B)}{(\Omega'^2 - 1)(\Omega'^2 - 1/B)}, \quad (5)$$

where $\Omega' = \Omega - K_{\parallel}V$.

In the limit of small density of beam, two space charge waves are degenerated into one mode which is described by $\Omega = K_{\parallel}V$, while the fast and the slow cyclotron waves are approximately described by $\Omega = K_{\parallel}V + 1$ and $\Omega = K_{\parallel}V - 1$, respectively.

It is expected that plasma wave (ion Bernstein wave) is coupled to these waves and instabilities may occur in an ion beam-plasma system. The dispersion relation (eq. (4)) is calculated for various plasma parameters, assuming that the wave number components $K_{\parallel}$ and $K_{\perp}$ are real and the frequency Ω is complex. The result for typical parameters is shown in Fig. 3, where the real and imaginary parts of Ω are plotted as functions of $K_{\parallel}$, with the dimensionless quantities

P , B , V , M , T and $K_{\perp}$ as parameters.

It is seen in Fig. 3(a), that mode coupling of plasma wave with beam waves does occur. Comparing this figure with Fig. 3(b), we note that the couplings with two fast waves of beam denoted by 1, 3, 4, 5 and 7 are stable ($\Omega_i < 0$) and the damping rate $-\Omega_i$ increases with the wave number component $K_{\parallel}$ increased. On the other hand, the coupling with slow space charge wave (denoted by 2) and with slow cyclotron wave (denoted by 6) are unstable ($\Omega_i > 0$) and growth rate Ω_i has several maxima near the intersections between dispersion curves of plasma and beam waves. Couplings with fast and slow space charge waves are degenerated in lower region of frequency.

3.3 Coupling with a slow space charge wave of beam

As described in a previous paragraph, coupling with fast waves of beam result in strong damping, so that they may be scarcely observed in an experimental study. On the other hand, couplings with slow waves result in unstable wave, so that the wave grows and its amplitude becomes large, which is the most dominant processes in a beam-plasma system. First, we will consider the coupling with slow space charge wave (denoted by 2 in Fig. 3). In Fig. 4, the structure of instability resulting from this coupling is shown for the typical value of plasma and beam parameters. The phase velocity of unstable wave $\Omega/K_{\parallel}$ is nearly equal to the beam velocity V , and the growth rate has several peaks in the frequency regions between the successive cyclotron harmonics ($n < \Omega < n+1$), that is, it has 'a cyclotron harmonic structure'. Here, the value of V is taken to be 9.5, which corresponds to $K_{\parallel}/\Omega \sim 9.5$. Therefore, the slow space charge wave couples with a neutralized ion Bernstein wave whose damping rate is fairly small, so that the instability does easily occur.

In Fig. 5, the equi-growth-rate-lines are shown on P - Ω space, which is determined by using the data shown in Fig. 4. The cyclotron harmonic structure

of growth rate is more obviously seen in this figure. The broken line shows the lower hybrid frequency ($\Omega_{LH}^2 = P+1$). When the plasma density is comparable with or larger than that corresponding to the lower hybrid frequency $P_0 = \Omega_{LH}^2 - 1$, the instability occurs practically.

In Fig. 6, the equi-growth-rate-lines are shown on $K_{||}-K_{\perp}$ space, for typical parameters. This figure may be considered as the relation between the growth rate and the propagation angle θ with respect to the direction of the magnetic field, i.e., $\tan\theta = K_{\perp}/K_{||}$. The cyclotron harmonic structure is seen obviously. Moreover, it is known that the growth rate has the maximum value at the proper angle θ , which is about 76° . This result suggests that the instability does not occur at the angle where the Landau damping is effective due to thermal electrons ($\theta \sim 90^\circ$) or thermal ions ($\theta \sim 0^\circ$) but occur where the neutralized ion Bernstein wave exists. This is consistent with the consideration in 3.1.

3.4 Coupling with slow cyclotron wave

Next, let's consider another coupling, that is, coupling with slow cyclotron wave, which also results in an unstable wave. In Fig. 7, the real and imaginary parts of frequency Ω are plotted as functions of $K_{||}$ with the quantities P , B , V , T , M and $K_{\perp}$ as parameters. In Fig. 8, the equi-growth-rate-lines are plotted in $P-\Omega$ space, by using Fig. 7 and many similar figures. The dotted line shows the lower hybrid frequency.

It is known that the growth rate has the cyclotron harmonic structure and the instability occurs practically for the plasma density P comparable with or larger than that corresponding to the lower hybrid frequency P_0 . This is quite same as the case of coupling with the slow space charge wave.

4 Summary

The behavior of the ion Bernstein wave is studied, by analyzing the dispersion

relation of the electrostatic wave.

In a Maxwellian plasma, the feature of propagation of the wave is varied as function of propagation angle θ . In the region of $K_{\parallel} \sim \Omega(MT)^{1/2}$ and $K_{\parallel} > \Omega$, the Landau damping due to the thermal electrons or the thermal ions is effective and the wave damps heavily, so that the wave can not appear. On the other hand, in the regions of $K_{\parallel} < \Omega(MT)^{1/2}$ and $\Omega(MT)^{1/2} < K_{\parallel} < \Omega$, resonant particles do not exist, so that the wave can propagate stably as the pure ion Bernstein wave in the former region and as the neutralized ion Bernstein wave in the latter region.

In an ion beam-plasma system, coupling with the slow space charge wave and the slow cyclotron wave of beam result in an unstable waves for the plasma density comparable with or larger than that corresponding to the lower hybrid frequency ($P > \Omega_{LH}^2 - 1$). The growth rate Ω_i has the cyclotron harmonic structure, that is, it has several maxima in the regions of $n < \Omega < n+1$. Moreover, Ω_i becomes maximum at the propagation angle θ where the beam wave couples with the neutralized ion Bernstein wave, because of small Landau damping.

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Figure Captions

- Fig. 1 Dispersion relation surface in a Maxwellian plasma in the frequency region of $1 < \Omega < 2$. (a) Frequency of the wave Ω . (b) Damping rate $-\Omega_i$.
- Fig. 2 Dispersion relation curves in a Maxwellian plasma in the frequency region of $2 < \Omega < 3$, with $K_{\perp}$ as a parameter.
- Fig. 3 Dispersion relation curve in an ion beam-plasma system. (a) Frequency of the wave Ω . (b) Damping or growth rate Ω_i . The coupling of plasma wave with the beam waves is obviously seen.
- Fig. 4 Dispersion relation of the unstable wave resulting from the coupling with the space charge wave of beam. It is seen that the growth rate Ω_i has the cyclotron harmonic structure.
- Fig. 5 The equi-growth-rate-lines of unstable wave resulting from the space charge wave of beam in P - Ω space, with Ω_i as a parameter. The dotted line shows the frequency of the lower hybrid resonance, $\Omega_{LH}^2 = P+1$.
- Fig. 6 The equi-growth-rate-lines of unstable wave resulting from the coupling with the space charge wave of beam in $K_{\parallel}$ - $K_{\perp}$ space, with Ω_i as a parameter. The maximum growth rate appears at the proper propagation angle θ , where the neutralized ion Bernstein wave couples with the beam wave.
- Fig. 7 Dispersion relation of the unstable wave resulting from the coupling with the slow cyclotron wave of beam.
- Fig. 8 The equi-growth-rate-lines of unstable wave resulting from the coupling with the slow cyclotron wave of beam in P - Ω space.

Fig. 1

Fig. 2

Fig. 3 (a)

Fig. 3 (b)

Fig. 4

Fig. 6

Fig. 7

Fig. 8