Absolute Instability of the Bernstein Wave in a Beam-Plasma System

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Absolute Instability of the Bernstein Wave in a Beam-Plasma System

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The backward wave with respect to the propagation component along the magnetic field is observed in a Maxwellian plasma, which can be explained consistently by the dispersion relation of the Bernstein wave. When an electron beam which is more intense than the threshold value is injected into the plasma, are observed a strongly excited wave whose frequency spectrum is very sharp (its normalized frequency width between half-power points $\Delta\omega/\omega$ is about one percent). From the comparison with the measurement of the amplification factor, it is shown that this spontaneously excited wave cannot be considered to be a manifestation of the thermal noise amplified due to the convective instability but self-oscillation due to the absolute instability. Moreover, the dependencies of this excitation of the wave on the plasma electron temperature T_e and the electron beam parameters (its density n_b and velocity v_b) are investigated in detail, the results of which are consistently explained by the theoretical consideration.

1. Introduction

In a recent few years, the instabilities of plasma wave in non-equilibrium plasmas (for example, bi-Maxwellian plasma and beam-plasma system) have been investigated with great interest, because of not only a physical significance of its study but also its development into nonlinear wave phenomenon and turbulent plasma heating effect. The instabilities are divided into two species; one of them is the convective instability which is concerned with a forward wave and the other is the absolute one which is concerned with a backward wave.[1]

The former has been studied already by many authors for various plasma waves (ion sound wave, electron plasma wave, electron Bernstein wave and so on) both theoretically and experimentally. However, the latter has been fully done theoretically[1] but not yet experimentally except a few instances.[2),3] On the other hand, the parametrically excited absolute instability is investigated from the viewpoint of

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the plasma heating by using the electromagnetic field of a high power laser.[4] Therefore, a study of the latter instability will be enhanced in the near future.

For an example, both instabilities are expected to occur in an electron beam-plasma system for the Bernstein wave, which appears as forward and/or backward waves dependently on the plasma parameters (the plasma density n_p, the electron temperature T_e and the electron cyclotron frequency ω_c). As described in our previous paper[5] (refered as I hereafter), the former instability occurs as the result of the amplification of a forward Bernstein wave in the system, while the phenomenon is never observed in a region of plasma parameters where a backward Bernstein wave is observed by the propagation experiment of the wave.

In this paper, we report that the self-oscillation is observed as a sharp frequency spectrum in the plasma parameter region described above and can be explained consistently as the results of the absolute instability of a backward Bernstein wave due to injection of an electron beam. In next section, the experimental apparatus and procedures are explained. In §3, the experimental results and discussions are described and in final section, the conclusions of the paper are noted in brief.

2. Experimental Apparatus and Procedures

An apparatus used here is the same as that used in our previous experiment about the convective instability of Bernstein wave reported in I. Let' s describe about both experimental apparatus and procedures in brief, because they have been explained in detail in I.

In order to study instabilities of waves in an electron beam-plasma system, the characteristics of the wave in a quiet plasma in thermal equilibrium must be examined and then, the behavior of them must be done in a beam-plasma system. Therefore, we set up the apparatus as shown in Fig. 1, in which a plasma and an electron beam are generated independently, so that the parameters of them can be varied independently. It is consisted of three regions, i. e., the dc discharge region, the plasma diffused region (or the region of beam-plasma system) and the beam-generated region. Ar gas is fed into the discharge region and, by using the method of differential pumping, the three regions denoted above are maintained at $1 \sim 2 \times 10^{-2}$ torr, $3 \sim 7 \times 10^{-4}$ and $0.8 \sim 1 \times 10^{-4}$

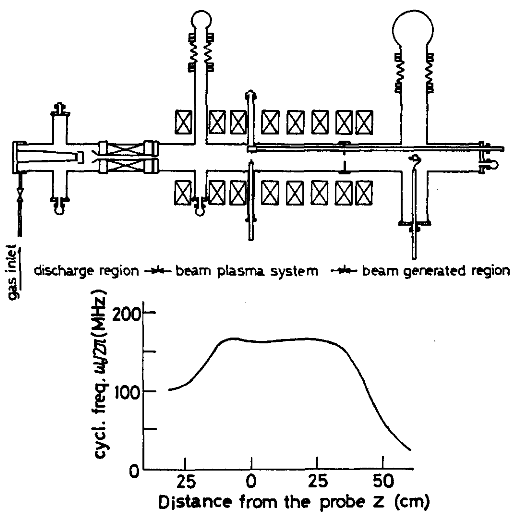

Fig. 1. The experimental apparatus and the distribution of a magnetic field intensity.

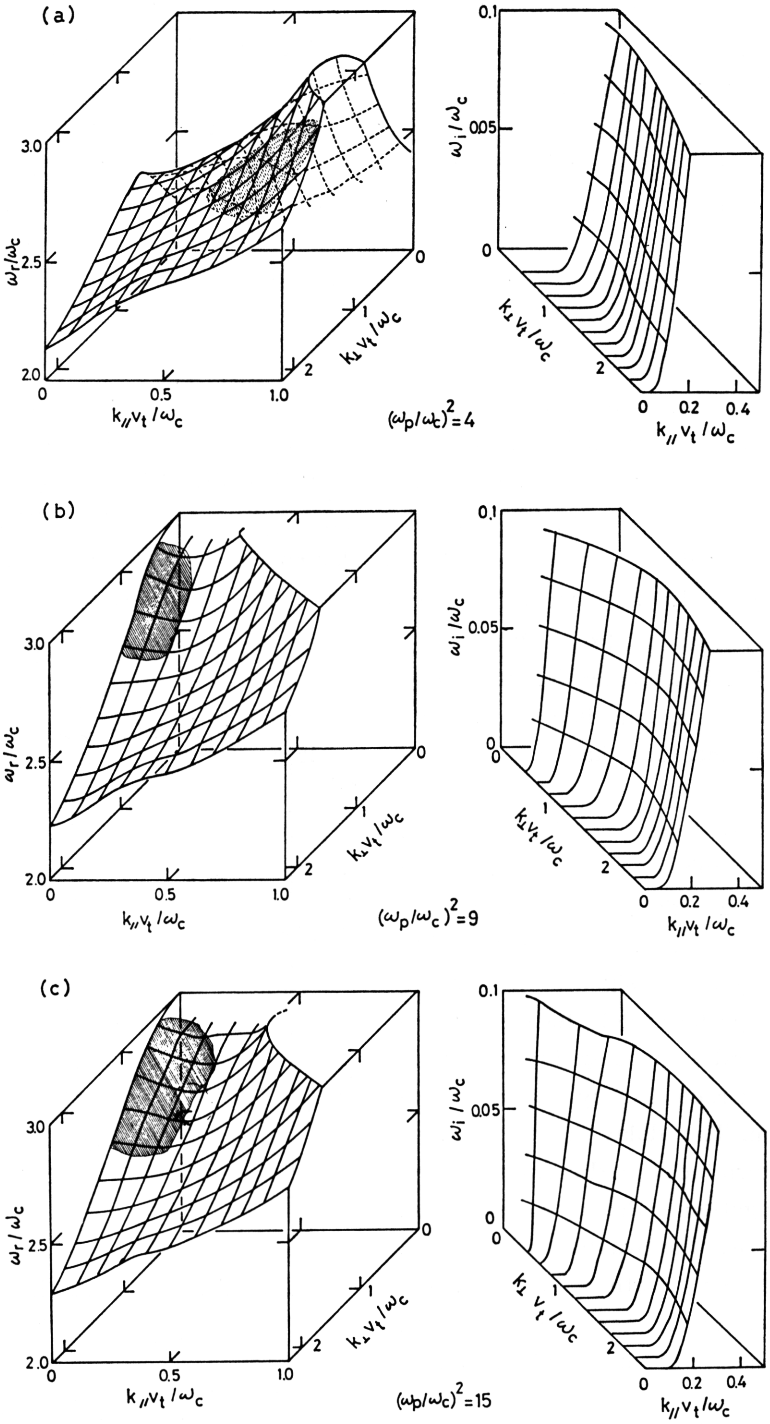

Fig. 2. The dispersion relation surfaces in ω-**k** spaces and ω_i-**k** ones, calculated by using eq. (1).
(a) $\omega_p^2/\omega_c^2=4$, (b) $\omega_p^2/\omega_c^2=9$ and (c) $\omega_p^2/\omega_c^2=15$. Hatched regions show the existing of the backward Bernstein wave.

torr, respectively. The plasma is produced by a dc discharge and diffused through an orifice and a hole at the center of an anode into the plasma diffused region along the external magnetic field, which is applied paralell to the axis of apparatus and whose intensity distribution is shown in Fig. 1. The field is uniform within 3 percent in the region and its strength is 60 gauss, which is corresponding to $\omega_c/2\pi=168$ MHz. The plasma is supported by the field near the axis of glass tube (95 mm in diameter and 700 mm in length), and its density profile in radial direction is shown in Fig. 3 in I, that in axial direction being approximately uniform. When the discharge current I_d is varied from 2 to 23 mA, the plasma density n_p is varied from 8×10^8 to 9×10^9 cm^{-3} but the electron temperature T_e is constant at about 6 ~10 eV in the region.

An electron beam is produced by the Pierce gun in the beam-generated region, and injected into the plasma-diffused region through a hole of 15 mm in diameter. When the accerelation voltage V_b of the beam is changed from 50 to 500 V, the current of the beam I_b changes from 0.18 to 3.1 mA, under normal operation. (The perveance of the gun being about 5×10^{-7} $AV^{-3/2}$.) The electron density of the beam n_b is varied from 1.5×10^8 to 4.5×10^8 cm^{-3}, but the temperature T_b of the beam is constant at about 0.3 eV. When n_b ($\propto I_b/\sqrt{V_b}$) and V_b must be varied independently, the heater current of electron gun is controled, so that the perveance is adjusted at the suitable value.

In order to excite and receive the wave, three coaxial probes are inserted in the plasma diffused region, one of them being movable radially and the others being movable axially. The signal of the waves excited by a probe is detected using another probe and its propagation pattern is measured and recorded by the interferometer system. The delay line is used in order to determine the direction of the wave propagation. From the recorded wave patterns, the wave number and damping rate (or growth rate) are determined. When an intense electron beam is injected and the wave is excited spontaneously, the self-correlation is measured by using the two probes of them. The excited and/or received frequency is varied from 168 MHz to 500 MHz. When the intensity of received signal is determined, it is compared with and equalized to that of the impulse generator by inserting a

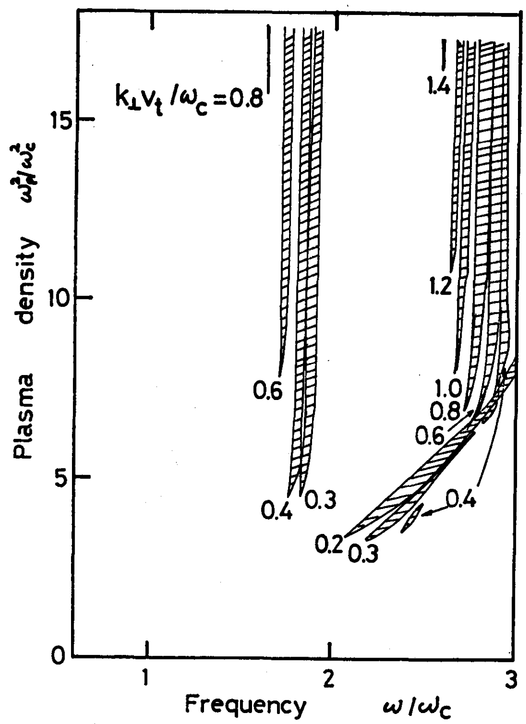

Fig. 3. The regions in n_p-ω space where the backward waves exist, with $k_\perp v_t/\omega_c$ as a parameter.

known value of attenuation in the transmission line from the plasma to the receiving system.

3. Experimental Results and Discussions

3.1 Observation of a backward wave with respect to the propagation component along the field and a theoretical consideration using the dispersion relation

In our previous paper[5] denoted by I, it is described that the forward and/or backward waves with respect to the propagation component along the field can appear in a thermal equilibrium plasma and the convective instability is observed for the former wave when an electron beam is injected. While, for the latter wave, the absolute instability may be expected to occur by injection of so intense electron beam that the growth factor of the wave does get over the damping one. It is the main subject in this paper.

In Fig. 4 (a) of the paper I, a feature of a backward wave propagation is shown, which is confined in an inner region for the upper hybrid layer ($\omega_p^2=\omega^2-\omega_c^2$, where ω_p is the plasma frequency) and can propagate obliquely to the field because of the rather small damping along the field by satisfying the condition of $|\omega-n\omega_c|\gg k_{\parallel}v_t$ for all integers n, where $k_{\parallel}$ and v_t is the wave number component along the field and the thermal velocity of plasma electrons. The result is explained consistently by using the dispersion relation surface of the wave, as shown in Fig. 6 (a) of I.

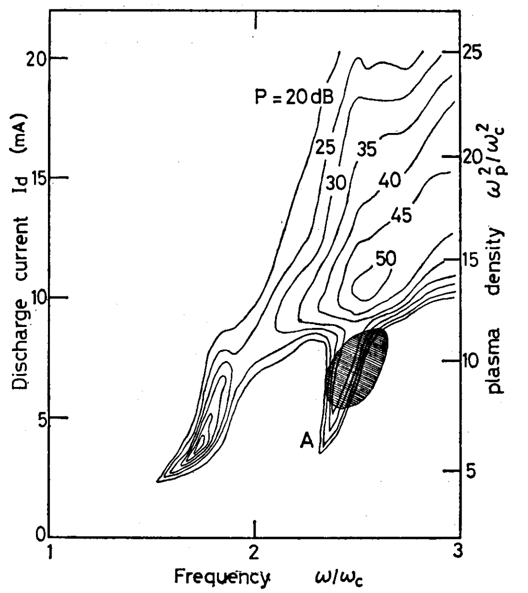

Fig. 4. The distribution of the spontaneously excited wave intensity. Solid curves show the equi-power lines and a hatched region does that where the backward wave exists. $V_b=250V$ and $I_b=3.1mA$.

As a diameter of plasma is much larger than a wave length as shown in Fig. 4 (a) of I, the plasma may be considered to be infinete for the wave, so that we can use a well-known dispersion relation,

$$K(\omega,k_{\parallel},k_{\perp})=1+\frac{\omega_p^2}{k^2v_t^2}\Big[1+\sum_{n=-\infty}^{\infty}\exp(-\lambda)I_n(\lambda)\frac{\omega}{\sqrt{2}\,k_{\parallel}v_t}Z\Big(\frac{\omega-n\omega_c}{\sqrt{2}\,k_{\parallel}v_t}\Big)\Big]=0, \qquad \cdots\cdots\cdots(1)$$

where $\lambda=(k\ v_t/\omega_c)^2$, I_n is the Bessel function of second kind, Z is the plasma dispersion function and k is the wave number component across the field. This equation can be calculated for various plasma parameter ω_p^2/ω_c^2, assuming that a frequency is complex $\omega+i\omega_i$ and a wave number is real k. Several results of the calculations are shown in Fig. 2. Hatched regions in figures show those where backward waves with respect to the propagation component along the field ($\partial\omega/\partial k_{\parallel} \cdot k_{\parallel} < 0$) can exist. In these regions, $k_{\parallel} v_t/\omega_c$ is much smaller than unit. Therefore, if ω/ω_c is nearly equal to half integer, the condition $|\omega - n\omega_c| \gg k_{\parallel} v_t$ is satisfied for all integers n and the damping rate ω_i of the wave is sufficiently small, which suggests that the absolute instability can occur due to the injection of an intense electron beam.

From many results of calculations of eq. (1) for various plasma parameters, we can obtain the regions in the parameter space ($n_p - \omega$ space) where the backward wave can exist, as shown in Fig. 3. The absolute instability can be expected in these regions.

3.2 Spontaneous excitation of the wave with the anomalously sharp frequency spectrum by injection of the intense electron beam

As described in I, when a rather intense electron beam is injected, spontaneous excitation of Bernstein waves are observed to occur and explained as a manifestation of the thermal noise of plasma amplified due to the convective instability of the wave, from comparison of the power distribution with the distribution of amplification factor determined from the wave propagation experiment. Injecting more intense electron beam, the excited power does not only increase extremely, but the excitation of a wave with anomalously sharp frequency spectrum is observed, whose normalized frequency width $\Delta\omega/\omega$ is about one percent. Its feature is shown in parameter space ($n_p - \omega$ space) (in Fig. 4), where solid curves show the equi-power lines and a sharp spectrum excitation described above is seen in the region denoted by 'A'. This spontaneous excitation does never appear in the case of more weak electron beam as shown in Fig. 13 of I and does appear suddenly when the electron beam intensity gets over a certain threshold value, whose feature is quite different from that shown in I. The hatched region in Fig. 4 does show that where the backward waves along the field are observed in the experiment of wave propagation (as shown in Fig. 4 (a) of I).

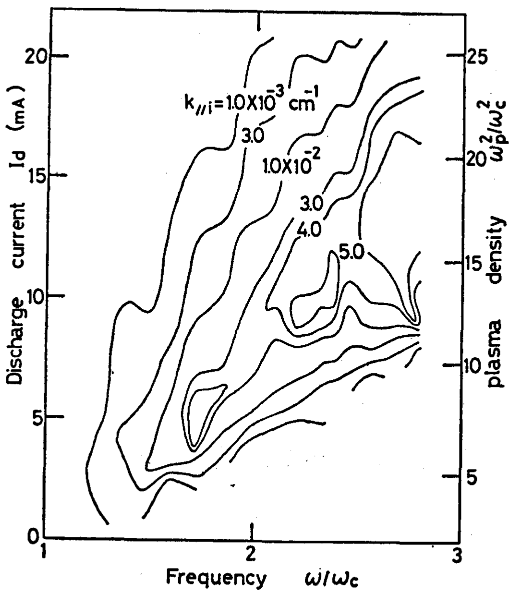

Fig. 5. The distribution of the amplification factor. Solid curves show the equi-amplification-factor lines. V_b=250V and I_b =3.1mA.

On the other hand, under the same experimental condition as the case of Fig. 4, the amplification factor $k_{\parallel i}$ is measured, by exciting the wave at the position near the beam inlet where a spontaneous excitation does not occur and recording the wave pattern propagating along the axis using the interferometer system. The result of which is shown in Fig. 5, where solid curves show the equi-amplification-factor lines. Comparing the figure with Fig. 4, the excited wave with broad frequency spectrum are explained as a manifestation of thermal noise amplified due to the convective instability as described in I but the wave with sharp spectrum denoted by 'A' can not be explained. These behaviours of the latter excitation of the wave suggest strongly that it is the self-oscillation due to the absolute instability of the backward Bernstein wave.

3.3 Dependency of the sharp spectrum excitation on the plasma and beam parameters

As described in 3.1 of this paper, the backward wave which enhances the absolute instability can only appear, when a parameter $k_{\parallel} v_t/\omega_c$ is smaller than a certain threshold value. On the other hand, from the observation of the self-correlation of the excited wave, it is known that the wave satisfies the Cherenkov excitation condition, i. e., $k_{\parallel} v_b \approx \omega$. Therefore, under the constant beam velocity v_b, the electron temperature T_e must be smaller than a threshold value, while under the constant T_e, v_b must be larger than a threshold value, in order to excite the absolute instability which concerns with the backward wave. Moreover, though the backward waves may exist, the absolute instability can not occur, when the growth factor of them does not get over the damping factor, so that the beam density n_b must be larger than the threshold value in order to excite the instability.

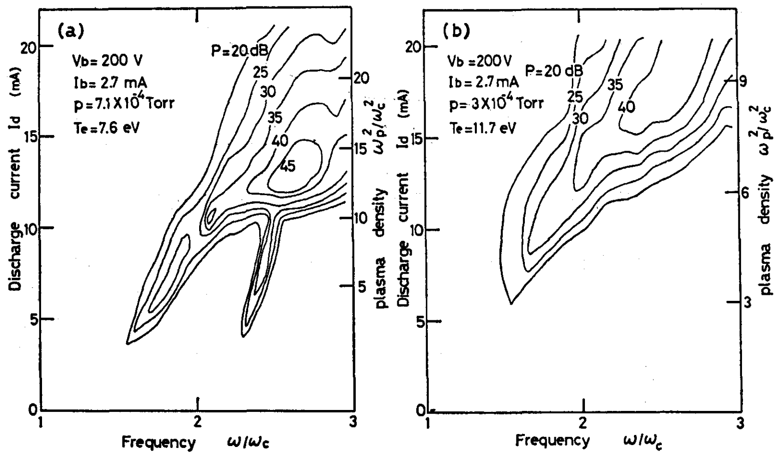

Fig. 6. The dependency of the appearence of a sharp frequency spectrum wave on the plasma electron temperature.
(a) The wave appears for $T_e=7.6$ eV which is smaller than the threshold value $T_{eo}=8.5$ eV. (b) It does not appear for $T_e=11.7$ eV. $V_b=200$ V and $I_b=2.7$ mA.

From the viewpoint to verify the above consideration experimentally, the dependency of appearence of the sharp spectrum excitation with the parameters T_e, v_b and n_b is investigated in detail. At first, under the constant beam parameters v_b and n_b, the dependency on T_e is studied, the result of which is shown in Fig. 6. It is seen in the figure that the excitation does appear when T_e is smaller than the threshold value T_{eo}= 8.5 eV which is calculated frɔm the dispersion relation shown in Fig. 7, while it does not when T_e is larger than the threshold value.

Next, under the constant value of T_e, the dependency on v_b and n_b is studied in detail, a few results of which are shown in Fig. 8. From comparison of Fig. 8 (a) with Fig. 8 (b), it is known that v_b must be larger than the threshold value, in order to obtain the excitation, while from comparison of Fig. 8 (a) with Fig. 8 (c), when n_b is larger than the threshold value, the excitation is observed. The similar experiments are done for various parameters n_b and v_b under constant plasma parameters T_e and n_p, the result of which is shown in Fig. 9, where the circles show the appearance of the excitatiɔn and the dark circles show the disappearence of it. The threshold for the appearence of the excitation in parameter space ($n_b - v_b$ space) is considered in the next paragraph.

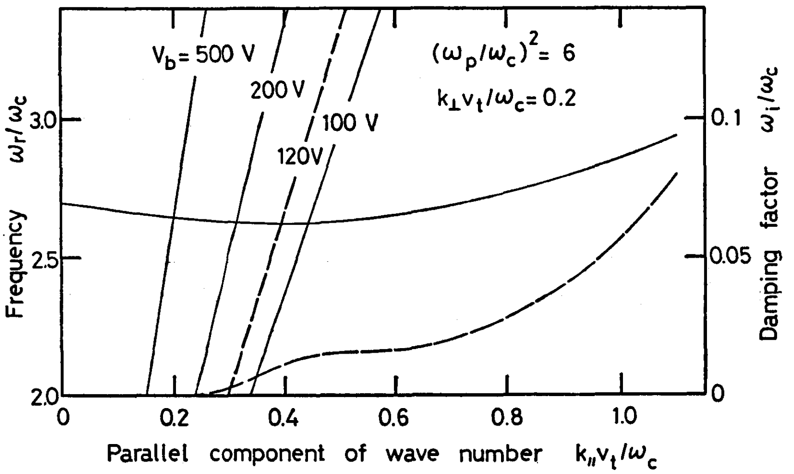

Fig. 7. The dispersion relation for $k_\perp v_t/\omega_c$=0.2 determined from the radial standing wavelength. Solid lines show beam dispersion relations for the various values of electron beam energy, when T_e=6eV. Broken line shows the damping rate ω_i/ω_c.

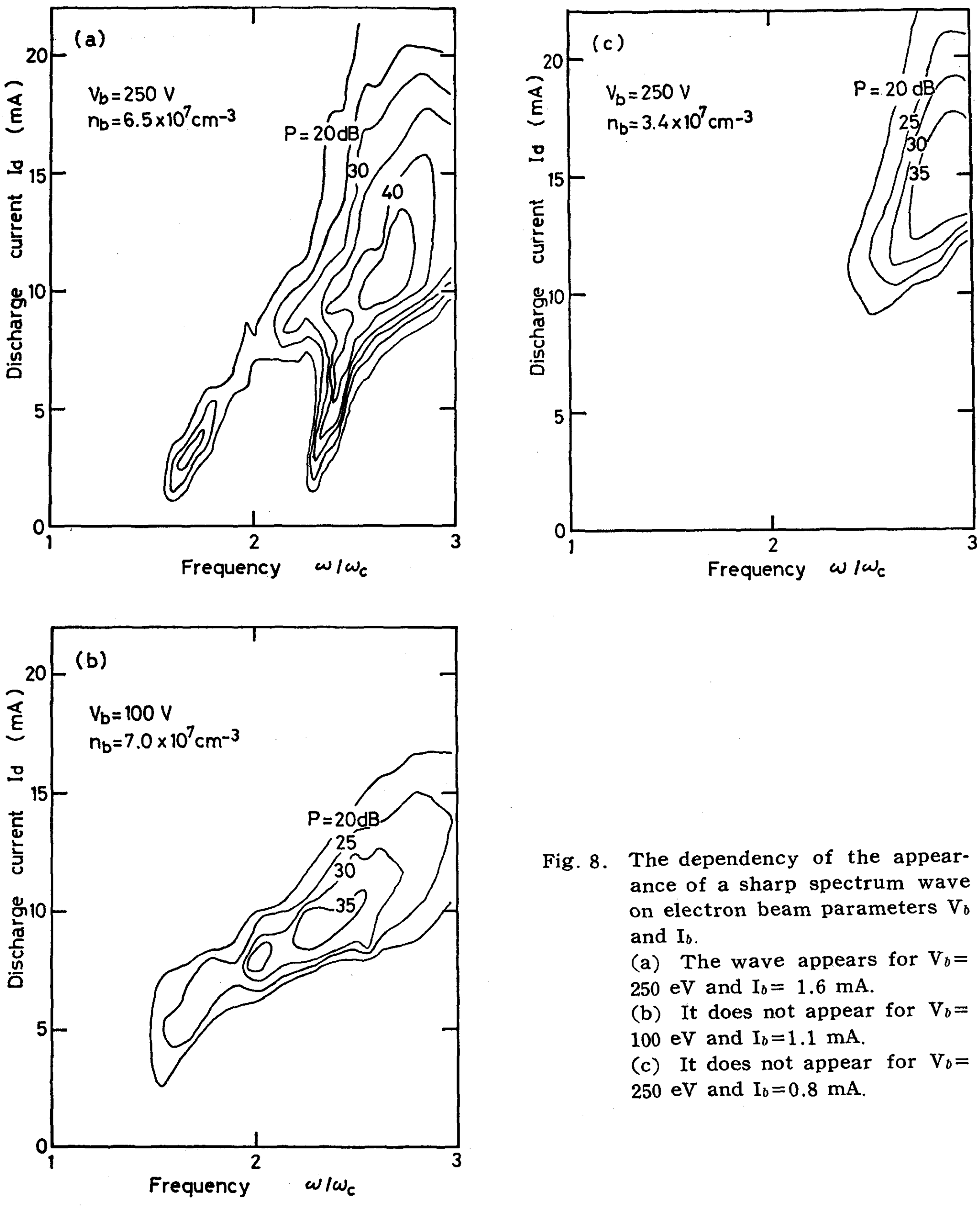

Fig. 8. The dependency of the appearance of a sharp spectrum wave on electron beam parameters V_b and I_b.
(a) The wave appears for V_b= 250 eV and I_b= 1.6 mA.
(b) It does not appear for V_b= 100 eV and I_b=1.1 mA.
(c) It does not appear for V_b= 250 eV and I_b=0.8 mA.

3.4 The consideration for a threshold value of n_b and v_b

Let's consider the threshold value in the parameter space ($n_b - v_b$ space) for occurence of the anomalous excitation of the wave, which is considered to be the results of an absolute instability of the backward Bernstein wave. For the plasma density of our plasma ($\omega_p^2/\omega_c^2=6$), the dispersion relation of the wave is shown in Fig. 7, where k $v_t/\omega_c=0.2\ cm^{-1}$ is determined from an observation of the radial standing wavelength $\lambda/2=0.9$ cm. As the wave excitation is the Cherenkov type one as described above, the absolute instability can not occur for a beam energy smaller than $V_{bo}=120$ eV, which corresponds to the beam velocity $v_b=6.4\times10^8$cm/sec and is shown by a dotted curve in Fig. 9. The instability is expected to occur for a beam energy larger than V_{bo} because a line of space charge wave of the beam intersects the dispersion relation line of plasma in the region of backward wave. If the growth rate gets over the damping rate in this case, the instability must occur practically. The condition is described as follows,[2]

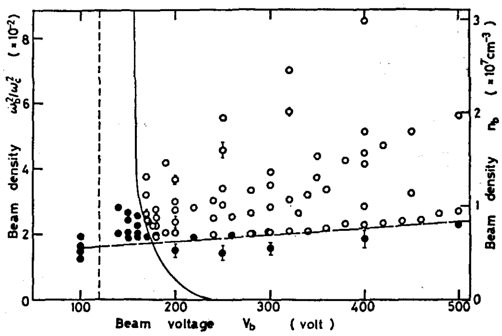

Fig. 9. The region in n_b-V_b space where the excitations are observed.

$$\frac{3\sqrt{3}}{2}\left(\frac{\alpha}{2}\right)^{2/3} B^{1/3} > D, \qquad \cdots\cdots\cdots(2)$$

where α is the ratio of the paralell group velocity component $v_{g\parallel}$ to the phase velocity component $v_{p\parallel}$ i. e., $\alpha = v_g/v_p$, $B^{1/3}$ is the growth factor of the wave and D is the normalized damping factor i. e., $D=\omega_i/\omega$. The expression of B is as follows,

$$B=\frac{\omega_b^2}{\omega^2}\frac{k^2 R}{\omega(\partial P/\partial\omega)}, \qquad \cdots\cdots\cdots(3)$$

ω_b is a plasma frequency of electron beam, $P=\mathrm{Re}\,(k^2\varepsilon_p)$, ε_p is a dielectric constant of plasma, R is the reduction factor which is described for the cylindrical system as follows,[1]

$$R=\frac{\pi^2}{4}(k_\perp b)^2\ N_0^2(k_\perp a), \qquad \cdots\cdots\cdots(4)$$

a and b are the plasma and beam radii, N_0 is the zeroth Neumann function.

Considering the Cherenkov excitation condition $v_{b\parallel}=\omega/k_\parallel$, the condition denoted by an inequality (2) is rewritten as follows,

$$\Omega_b^2>4\left(\frac{2}{3\sqrt{3}}\right)^3\frac{\Omega_i^3}{R}\left(\frac{\partial\varepsilon_p}{\partial\Omega}\right)_r\frac{1+(K_\perp/K_\parallel)^2}{(\partial\Omega/\partial K_\parallel)^2}\left(\frac{\Omega}{K_\parallel}\right)^2, \qquad \cdots\cdots\cdots(5)$$

where $\Omega_b=\omega_b/\omega_c$, $\Omega=\omega/\omega_c$, $\Omega_i=\omega_i/\omega_c$, $K_\parallel$ and $K_\perp$ are paralell and perpendicular components of the vector $K=kv_t/\omega_c$ and $(X)_r$ shows the real part of X. The result of calculation for threshold using an inequality (5) is shown by a solid curve in Fig. 9, which explains the experimental result for rather small beam voltage ($V_b<180$ V) but can not do for large beam voltage.

For the latter case ($V_b>180$ V), as $K_\parallel$ takes a small value which is determined by Cherenkov excitation condition, so that the damping factor D is very small as shown in Fig. 7, it is considered that the threshold value is much smaller than the former case ($V_b<180$ V).

In these circumstances, the other unknown mechanism which disturbs the self-oscillation due to the instability may be considered, for example, the wave energy loss of the system (the apparatus and plasma system). Using the well-known notion of Q-value, we may express the effective damping factor corresponding to this mechanism D_{eff} as follows,

$$Q=\frac{\omega}{\Delta\omega}=\frac{\omega E}{\Delta E}=\frac{2\pi}{1-\exp(2\omega_i T)}=\frac{2\pi}{1-\exp(4\pi D_{eff})}, \qquad \cdots\cdots\cdots(6)$$

where E is the total wave energy in the whole spectrum, ΔE is the energy loss from the whole spectrum per unit time and $T=2\pi/\omega$. Using the threshold value for Ω_b which is calculated from the broken line in Fig. 9, we can estimate the value of the effective damping factor $\Omega_{i\ eff}$ correponding to the wave energy loss, from the condition described by an inequality (5). As D_{eff} $(=\Omega_{i\ eff}/\Omega)$ can be determined in this way, Q-value is calculated by eq. (6). The results of calculations are shown in Table 1 with the experimentally determined Q-value from the normalized frequency width $\Delta\omega/\omega$ of the observed frequency spectrum. Both values, the Q-value calculated from the threshold value of Ω_b and that determined from the shape of the frequency spectrum, coincide within factor 3, which suggests that the threshold value in n_b-v_b space for the latter case ($V_b>$ 180 V) is dominated by rather the wave energy loss of the apparatus and plasma system than the damping factor of the wave.

Table 1.

V_b(V)	Calculated value		Experimental value	
	Ω_i	Q	$\Delta\omega/\omega$(%)	Q
200	6.7×10^{-8}	193	0.8−1.9	125−53
250	6.9	187	0.9−2.3	111−44
300	7.2	182	1.4−2.1	71−48
350	7.7	170	0.9−2.0	111−50
400	8.0	162	1.4−2.0	71−50
450	8.4	154	1.1−2.1	91−48

Moreover, because the analysis described above can explain consistently the experimental results, the assumption that the anomalous excitation of the wave is a self-oscillation due to the absolute instability of backward Bernstein wave, is considered to be correct.

4. Conclusion

Let's describe the results obtained experimentally and considered theoretically in this paper. The experiment on the propagation of the Bernstein wave shows that the forward wave along the field can not only propagate but also the backward wave can do. The existence of the latter suggests that the absolute instability can occur when an electron beam is injected. Practically, when the intense electron beam is injected, an excitation of the wave with an anomalously sharp spectrum is observed and it can be explained as the self-oscillation due to the occurence of the absolute instability of the Bernstein wave from its characteristics as follows;

1) The excitation does occur only in the region in $n_p-\omega$ space, where the backward Bernstein wave is observed in the experiment of the wave propagation.

2) When the electron temperature of plasma T_e becomes larger than the threshold value, it does disappear, which is explained from the dispersion relation of the wave.
3) For a rather low energy of electron beam (V_b <180 V in our experiment), it is excited when v_b and n_b becomes larger than the threshold value determined from the condition that the growth factor gets over the damping factor of the wave, which are estimated under the assumption of absolute instability.
4) However, for a higher energy of electron beam (V_b> 180 V), as the damping factor of the wave itself is much smaller and can be neglected, the excitation is dominated by the condition that the growth factor gets over the effective damping factor D_{eff} corresponding to the wave energy loss of the apparatus and plasma system. D_{eff} is nearly equal to that obtained from the Q-value of excited frequency spectrum.

Acknowledgement

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