Efficiency and Optimum DC Bias Field for Second Harmonic Generation due to Hot Electrons In Semiconductors

Efficiency and Optimum DC Bias Field for Second Harmonic Generation due to Hot Electrons in Semiconductors

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The efficiency of the yield of second harmonics in nonpolar nondegenerate semiconductors is estimated by a simple method without using the Boltzmann transport equation. For n–Ge and n–Si, the efficiency of 4 to 5 percent is obtained for the optimum dc bias field, about V_0/μ_0, i.e., the ratio of the saturation velocity to the low–field mobility. In order to give this estimation, a hyperbolic equation is assumed, which may describe the nonlinear velocity–electric field relation from the ohmic region up to the saturation region in the temperature range between 77 and 300K. The velocity–field characteristic by this approximation is found to be in good agreement with the experiment. It is suggested that the optimum field for n–Ge is lower than that for n–Si, and that the optimum bias field in both n–Ge and n–Si is decreased in the temperature range of interest as the temperature is lowered.

1 Introduction

The relation between current density and applied field becomes nonlinear due to hot electrons when a high electric field is applied across a semiconductor slab [1]. Recently the second harmonic generation (SHG) due to this nonlinearity has been demonstrated in the current density of nonpolar nondegenerate semiconductors by several authors [2]–[4] when a microwave field and a high dc bias field are applied simultaneously. Analytically the Boltzmann transport equation leads to an expression for the second harmonic component in the current density after cumbersome procedures. In this treatment we can estimate the yield of SHG and its optimum bias field by a simple method.

It can be expected that the expression of the drift velocity to the bias field will lead to the expressions of higher harmonics due to the nonlinearity in the current density. In Sec. 2, a hyperbolic equation is assumed, which has two asymptotes, $V=\mu_0 E$ and $V=V_0$, and may describe the nonlinear velocity-electric field relation from the ohmic region up to the saturation region, and this hyperbola is compared with experiment [5] in the velocity–field characteristic. In Sec. 3, the efficiency of SHG and its optimum bias field are calculated using this hyperbolic equation. The typical examples are carried out for n–Ge and n–Si. In Sec. 4, we briefly summarize the conclusions drawn in this paper.

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Since the acoustic phonon, nonpolar optical phonon, and intervalley scattering are assumed to be the dominant type of scattering in the nonpolar nondegenerate semiconductors in the temperature range between 77 and 300K, it may be considered that the collision frequency will give an upper limitation of frequency of the applied ac field.

2 A hyperbolic equation giving the nonlinear velocity-electric field relation

The velocity-field characteristic from the ohmic region up to the saturation region in n-Ge or n-Si in the temperature range between 77 and 300K shows a hyperbolic curve intuitively. The asymptotes may be considered to be the ohmic line and the saturation velocity line, i.e., $V=\mu_0 E$ and $V=V_0$. The carrier velocity-field relation was assumed to be given by the following equation [6]:

$$(V-V_0)(V-\mu_0 E-\beta V_0)=\alpha, \qquad \cdots\cdots\cdots(1)$$

where, V : carrier velocity
V_0 : saturation velocity
μ_0 : low-field mobility
E : electric field
α : hyperbolic parameter
β : intersectional coefficient.

The differential mobility dV/dE at zero field is given by eq. (1),

$$\left.\frac{dV}{dE}\right|_{E=0}=\frac{\mu_0}{1+\beta}. \qquad \cdots\cdots\cdots(2)$$

Eq. (2) shows that the mobility becomes $\mu_0/(1+\beta)$ near zero field. However, the exact low-field mobility is μ_0, and so a better approximation can be expected in the low field region using $\mu_0(1+\beta)$ instead of μ_0.

In this paper, in order to give the estimation of SHG, we assume that the velocity-field relation in n-Ge or n-Si is given by the following corrected equation.

$$(V-V_0)[V-\mu_0(1+\gamma)E-\beta V_0]=\alpha, \qquad \cdots\cdots\cdots(3)$$

where, γ : corrective coefficient.

Eq. (3) gives the drift velocity and the differential mobility of carrier respectively,

$$V=\frac{1}{2}[\{\mu_0(1+\gamma)E+V_0(1+\beta)\}-\{[\mu_0(1+\gamma)E-V_0(1-\beta)]^2+4\alpha\}^{1/2}], \qquad \cdots\cdots(4)$$

$$\frac{dV}{dE}=\mu_0(1+\gamma)(V_0-V)/[\mu_0(1+\gamma)E+V_0(1+\beta)-2V]\geqq 0. \qquad \cdots\cdots\cdots(5)$$

The variation of drift velocity with electric field is given by eqs. (4) and (5) in Table 1.

Drift velocity versus electric field by eq. (4) is shown in Fig. 1 and Fig. 2 for n-Ge and n-Si respectively when $\alpha=(1/10)V_0^2$, $(1/20)V_0^2$ and $(1/30)V_0^2$, and $\beta=\gamma=1/20$ and 1/10, and $\mu_0=3900\text{cm}^2/\text{V}\cdot\text{sec}$, $V_0=6.0\times10^6\text{cm/sec}$ in n-Ge and $\mu_0=1500\text{cm}^2/\text{V}\cdot\text{sec}$, $V_0=1.0\times10^7\text{cm/sec}$ in n-Si at 300K. The experimental curve [5] is shown simultaneously. It can be seen from Figs. 1 and 2 that eq. (4) gives good agreement with experiment qualitatively.

Table 1 The variation of drift velocity with electric field

E	0	$\frac{V_0}{\mu_0}\cdot\frac{1-\beta}{1+\gamma}$	∞
$\frac{dV}{dE}$	$\simeq\mu_0\frac{1+\gamma}{1+\beta}$	$\frac{\mu_0}{2}(1+\gamma)$ monotonous decrease	0
V	$\simeq 0$	$V_0-\alpha^{1/2}$ monotonous increase	V_0

$V=0$ and $\frac{dV}{dE}=\mu_0(1+\gamma)/(1+\beta)$ at $E=0$ when $\alpha=\beta V_0^2$.

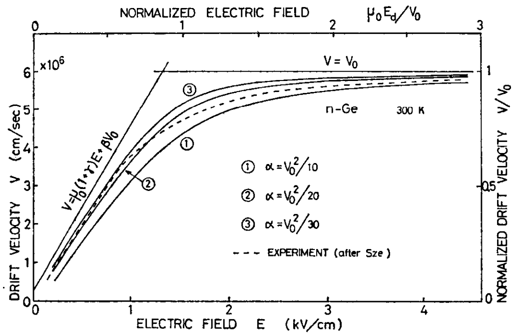

Fig. 1 (a)

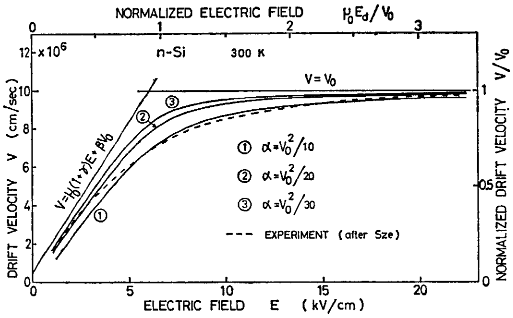

Fig. 2 (a)

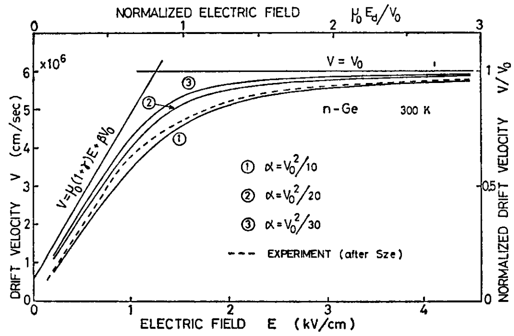

Fig. 1 (b)

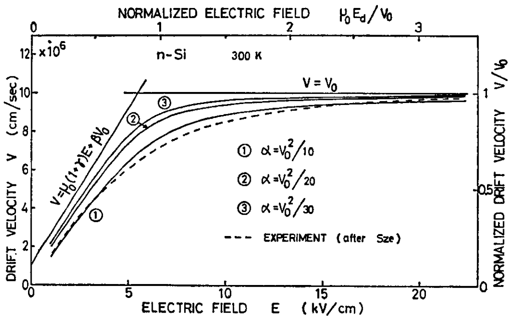

Fig. 2 (b)

Fig. 1. Drift velocity versus electric field intensity for n-Ge at 300K. (a) $\beta=\gamma=1/20$, (b) $\beta=\gamma=1/10$.

Fig. 2. Drift velocity versus electric field intensity for n-Si at 300K. (a) $\beta=\gamma=1/20$, (b) $\beta=\gamma=1/10$.

3 SHG in the current density

When a dc bias field E_d and E_1 a microwave field with frequency ω are applied to a semiconductor slab in the same direction simultaneously, the total applied electric field E can be written by

$$E=E_d+E_1\exp(-j\omega t), \qquad \cdots\cdots\cdots(6)$$

where $E_d \gg E_1$ and $\omega>0$. The current density J can be expanded in terms of higher harmonics due to nonlinearity as follows,

$$J = neV = J_0 + J_1 \exp(-j\omega t) + J_2 \exp(-2j\omega t) + \cdots\cdots, \qquad \cdots\cdots\cdots(7)$$

where n is the carrier concentration and e is the electronic charge. From eqs. (4), (6) and (7), the second component J_2 is given by

$$J_2 = -nea\mu_0^2(1+\gamma)^2 E_1^2 / [\{\mu_0(1+\gamma)E_d - V_0(1-\beta)\}^2 + 4\alpha]^{3/2}. \qquad \cdots\cdots\cdots(8)$$

From eq. (8) the efficiency η is given by

$$\eta = \left| \frac{J_2}{J_1} \right| = \alpha\mu_0(1+\gamma)E_1 / [\{\mu_0(1+\gamma)E_d - V_0(1-\beta)\}^2 + 4\alpha]^{3/2}, \qquad \cdots\cdots\cdots(9)$$

where $|J_1| = ne\mu_0(1+\gamma)E_1$. The efficiency η versus the normalized electric field $\mu_0 E_d/V_0$ is shown in Fig. 3 when $E_1 = (1/10)E_d$. Figs. 1, 2 and 3 show that η is 4 to 5 percent in n-Ge and n-Si, and the optimum dc bias field is about V_0/μ_0. Generally η is increased as E_1/E_d is increased. Analytically, we can see from eq. (9) that η has a maximum value at $E_d = V_0(1-\beta)/[\mu_0(1+\gamma)] \simeq V_0(1-\beta-\gamma)/\mu_0$ if E_1 is constant. Since $\beta \ll 1$ and $\gamma \ll 1$, the optimum bias field is about V_0/μ_0, which corresponds to the critical field at which two asymptotes intersect.

It is easily seen that no second harmonic is generated in the constant mobility region and in the constant velocity region whose characteristics are shown by the two asymptotes. This shows that the optimum field for SHG may exist. At this critical field, the differential mobility is $\mu_0/2$ and the curvature of the velocity-field characteristic is almost maximum. Therefore, it is understood that the yield of second harmonics may have a maximum value at this field.

At 77K from the experiment [7] and [8], the low-field mobility and the saturation velocity are as follows, $\mu_0 = 20000 \text{cm}^2/\text{V}\cdot\text{sec}$ and $V_0 = 1.0 \times 10^7 \text{cm/sec}$ in n-Ge, and $\mu_0 = 21000 \text{cm}^2/\text{V}\cdot\text{sec}$ and $V_0 = 1.5 \times 10^7 \text{cm/sec}$ in n-Si. The optimum dc bias field is

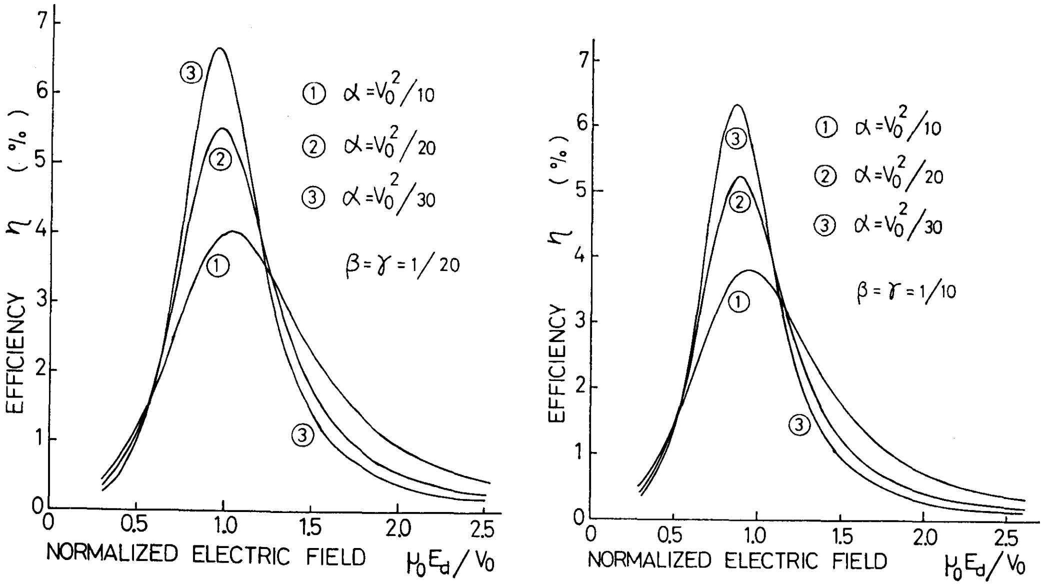

Fig. 3 (a) Fig. 3 (b)

Fig. 3. Efficiency of SHG versus normalized electric field intensity.

given respectively, 5.0×10^2 V/cm for n–Ge and 7.1×10^2 V/cm for n–Si if the optimum field is given by V_0/μ_0.

In the temperature range between 77 and 300K, the optimum bias field is decreased as the temperature is lowered, which means that the lower temperature is favourable in the yield of second harmonics. It is suggested that n–Ge is more favourable than n–Si as the former has the lower optimum field than the latter.

4 Conclusions

The efficiency of the yield of second harmonics and the optimum bias field in nonpolar nondegenerate semiconductors are estimated. In this estimation a hyperbolic equation is assumed to describe the nonlinear velocity–electric field relation. This equation shows good agreement with experiment qualitatively, and leads to the calculation about SHG simply. Although eq. (4) is not compared with the experiment at 77K, the efficiency of SHG at 77K is considered to be almost equal to the value at 300K.

It is concluded for n–Ge and n–Si in the temperature range between 77 and 300K as follows :

1) The efficiency of SHG to the fundamental wave is 4 to 5 percent.
2) The yield of second harmonics and the efficiency of SHG have the maximum value respectively for the optimum bias field, about V_0/μ_0.
3) The optimum field is decreased as the temperature is lowered.
4) The optimum field for n–Ge is lower than that for n–Si.
5) The eq. (3) (or (4)) gives a good approximation for the nonlinear velocity-field relation by using the suitable values of α, β and γ.
   For example, $\alpha=(1/10)V_0^2$, $\beta=1/10$ and $\gamma=1/10$ are suitable values to be fit for the experimental deta [5].

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References

1) E. M. Conwell, High field transport in semiconductors.
New York : Academic Press, 1967.
2) P. K. Kaw, J. Appl. Phys. **40,** 793 (1969).
3) P. Das, J. Appl. Phys. **40,** 4206 (1969).
4) M. S. Sodha and B. M. Gupta, Phys. Stat. Sol. **37,** 311 (1970).
5) S. M. Sze, Physics of semiconductor devices.
New York : John Wiley and Sons, 1969. p.59.
; C. B. Norris and J. F. Gibbons, IEEE Trans. Electron Devices, **ED-14**, 38 (1967).
6) H. Yamamoto and H. Iwasawa, Proc. IEEE **61,** April, 1973.
7) E. J. Ryder, Phys. Rev. **90,** 766 (1953).
8) Michele Costato and Lino Reggiani, Phys. Rev. **B3,** 1501 (1971).
; C. Canali, G. Ottaviani and A. Alberigi Quaranta, J. Phys. Chem. Solids (to be published).